Mobility Analysis of a Novel 3-5R Parallel Mechanism Family

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Mobility analysis of a novel 3-5R parallel mechanism family whose limb consists of a 2R and a 3R parallel subchain is performed by the aid of screw theory. A mobility criterion applicable to such 3-leg parallel mechanisms in which each kinematic chain contains five kinematic pairs is proposed. It is shown that under different structural conditions, the 3-5R parallel mechanism can have 3, 4, or 5 DOF (degrees of freedom). The structural conditions that guarantee the full-cycle mobility are analyzed. The analysis and the method presented in this paper will be helpful in using such a 3-5R parallel mechanism family and introduce new insights into the mobility analysis of parallel mechanisms. [DOI: 10.1115/1.1637651]

Introduction

With the development of parallel robot technology, simpler and less expensive spatial lower-mobility parallel mechanisms have drawn a lot of interests. The lower-mobility parallel mechanism has fewer than six DOF, which are suitable for many tasks that do not require all the six DOF.

An important one widely studied is the 3-RPS parallel robot [1–5] which has two rotational degrees of freedom and one translational degree of freedom, where *S* denotes the spherical joint, *R* denotes the revolute pair, and *P* denotes the prismatic pair. Cox and Tesar [6], studied a 3-DOF spherical mechanism. Gosselin and Angeles proposed an optimum kinematic design of a planar 3-DOF parallel manipulator and a spherical 3-DOF parallel manipulator [7–8]. Clavel proposed the successful 3-DOF translational parallel mechanism, DELTA robot [9]. Tsai proposed some 3-DOF translational parallel mechanisms in his patent [10], one of which is the 3-RRC parallel mechanism.

In 1999, Rolland proposed two 4-DOF parallel mechanisms, the Manta and the Kanuk [11], for material handling, which use parallelograms to eliminate rotations. In 2000, Zhao and Huang proposed a 4-DOF 4-URU parallel mechanism [12], where U denotes a universal joint. In 2001, Zlatanov and Gosselin came up with a 4-DOF parallel mechanism [13]. In 2002, Huang and Li invented two 4-DOF and two 5-DOF parallel mechanisms [14].

In a lower-mobility parallel mechanisms, limbs exerts some structural constraints on the moving platform and the combination of all the limb constraints determines what DOF of the moving platform are constrained. Thus, the structural condition of the limb kinematic chain itself and the structural condition of all the limbs basically determine the mobility of the mechanism. For example, under different structural conditions, the 3-UPU parallel mechanism can be a 3-DOF translational mechanism [15,16], or a 3-DOF rotational mechanism [17], or a 4-DOF mechanism [18] and a 5-DOF instantaneous mechanism [19].

In this paper, we performed the mobility analysis of a novel 3-5R parallel mechanism family whose limb consists of a 2R and a 3R parallel subchain based on constraint analysis. It is shown that the 3-5R parallel mechanism may have 3, 4, or 5 DOF under different structural conditions.

1 Theoretical Fundamentals of Constraint Analysis

In screw theory [20,21], the unit screw associated with a revolute pair or representing a force is given by $\$=(\boldsymbol{s};\boldsymbol{r}\times\boldsymbol{s})=(l\ m\ n;a\ b\ c)$, where $\boldsymbol{s}$ is a unit vector along the screw axis, $\boldsymbol{r}$ is the position vector of any point on the screw axis, *l*, *m*, *n* denote three direction consines. The unit screw associated with a prismatic pair or representing a couple is given by $\$=(0;\boldsymbol{s})=(0\ 0\ 0;l\ m\ n)$.

The structure constraint acting on the moving platform by limbs can be represented by wrenches, which could be forces, couples or wrenches. We define the wrenches acting on the moving platform by a single limb as the limb structure constraint screws, which constitute the *limb constraint system*. And all the limb structure constraint screws constitute the *mechanism constraint system*, which represents the combined effect of all limb structural constraints and determines the mobility of the mechanism. The maximum linearly independent number of the mechanism constraint system equals the constrained DOF of the parallel mechanism. Based on linear dependency of the limb structural constraints under different geometrical conditions, we can obtain the mechanism constraint system as well as the constrained DOF of the moving platform [22]. Reference [22] also proposed two tables describing the linear dependency of the limb constraints and the constrained DOF of the moving platforms. Both the number and property of mobility can be judged according to the mechanism constraint system.

It must be pointed out that the mechanism constraint system is instantaneous. When the mechanism moves, the mechanism constraint system may change, consequently, the mobility will change. When analyzing the mobility, we must identify whether the mechanism constraint system will remain the same one after the moving platform undergoes any finite feasible motion. Here the same one does not necessarily mean that the mechanism constraint system must be in the same form as the original one, but the mechanism constraint system exerts the same constraint on the moving platform as the original mechanism constraint system. This can be detected by examining the standard base or base of the mechanism constraint system, because the specific screws are the linear combination of a group of screws bases. If the mechanism constraint system changes after the moving platform undergoes any finite feasible motion, we call this mechanism an instantaneous mechanism.

The mobility can also be given by the general Grübler criterion

$$M=d(n-g-1)+\sum_{i=1}^{g} f_i, \tag{1}$$

where *M* denotes the mobility of the mechanism, *d* the order of the mechanism, *n* the number of links, *g* the number of kinematic pairs, and f_i the freedom of the *i*th pair.

The general Grübler-Kutzbach criterion is of importance in mobility analysis of lower-mobility parallel mechanism. Because most of the lower-mobility parallel mechanisms are over-

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constrained mechanisms, it is necessary to take the common constraints and the redundant constraints into consideration. Thus, we rewrite the Grübler-Kutzbach criterion as

$$M=d(n-g-1)+\sum_{i=1}^{g} f_i+\nu, \tag{2}$$

where ν denotes the number of redundant constraints.

The order of a mechanism is given by

$$d=6-\lambda, \tag{3}$$

where λ is the number of the common constraints.

Because the limb twist system of the 3-5R parallel mechanism is a 5-system, each limb kinematic chain only exerts one structural constraint on the moving platform. The three limbs exert three limb constraints on the moving platforms in total. The three limb constraints form the mechanism constraint system, which must be a $6-M$ system in the general configuration.

The common constraint can be defined as a screw reciprocal to all unit twists associated with kinematic pairs in a lower-mobility parallel mechanism [22,23]. In terms of geometry, a common constraint exists if and only if all the three limb constraints are coaxial, namely, they form a 1-system. In this case, when the common constraint exists, there are no redundant constraints, namely, $\nu=0$.

When there are no common constraints, if the three limb constraints are linearly dependent and form a $l-$ system $(l\leqslant 3)$, there exist redundant constraints and we have

$$\nu=3-l \tag{4}$$

It should be noted that the three limb constraints should equal the sum of the $6-M$ mechanism constraints, the limb constraints which form the common constraints, and the redundant constraints. Considering that each common constraint constrains one DOF of the moving platform, we have.

$$3=(6-M-\lambda)+3\lambda+\nu. \tag{5}$$

Thus, in this case, the mobility can also be given by

$$M=3+2\lambda+\nu. \tag{6}$$

It should be noted that Eqs. (4) and (6) are applicable to any symmetrical 3-leg parallel mechanism in which each kinematic chain contains five kinematic pairs. For example [19], for Tsai's 3-DOF 3-UPU translational parallel mechanism, we have $\lambda=0$ and $\nu=0$. Using Eq. (6), we have $M=3$; for Huang's 4-DOF 3-UPU parallel mechanism, we have $\lambda=0$ and $\nu=1$. Using Eq. (6), we have $M=4$; for the instantaneous 5-DOF 3-UPU parallel mechanism, we have $\lambda=1$ and $\nu=0$. Using Eq. (6), we have $M=5$.

Another example is the general 3-RPS parallel mechanism [1] in which each RPS limb exerts a constraint force on the moving platform. The constraint force passes through the center of the spherical pair and is parallel to the first revolute axis. The three limb constraint forces form a triangle parallel to the base plane, which guarantees that they are linearly independent and form a 3-system [24]. Thus, we have $\lambda=0$ and $\nu=0$. Using equation (6), we have $M=3$.

2 Mobility Analysis

In what follows, we always assume that the moving platform is parallel to the base plane in the initial configuration. For convenience, $\$_{ij}$ is used to represent the unit twist associated with the j^{th} kinematic pair in the i^{th} limb; $\$_{ij}^r$ is used to represent the j^{th} unit wrench exerted by the i^{th} limb; $\$_{mj}$ is used to represent the j^{th} unit twist in the mechanism twist system; $\$_{mj}^r$ is used to represent the j^{th} unit wrench in the mechanism constraint system.

2.1 A 3-DOF Translational 3-5R Parallel Mechanism. A 3-DOF translational 3-5R parallel mechanism is shown in Fig. 1,

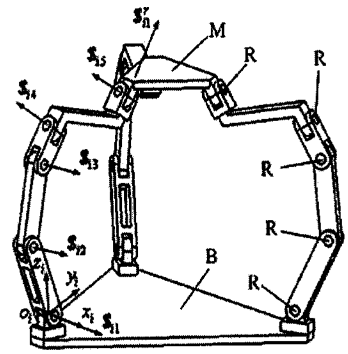

Fig. 1 3-$^{b}\underline{RRR}\overline{RR}$ parallel mechanism

where M denotes the moving platform and B denotes the base. In each limb, counting from the base, the first three revolute axes are parallel to the base plane and form a 3R parallel subchain, denoted by $^{b}\underline{RRR}$, where the superscript b denotes the axis of the pair is parallel to the base plane and the underline denotes that the three successive revolute axes are parallel. The following two revolute axes are parallel and form a 2R parallel subchain. The axes of the 2R parallel subchain bevel the moving platform as well as the base plane. The three limbs are set symmetrically, that is, the axis of the grounded revolute pair is not parallel to one another. Such a mechanism was presented independently by Sugimoto and Hara [25,26], Frisoli et al. [27] and Carricato and Parenti-Castelli [28].

Set the central point of the first revolute pair as the origin of the i^{th} limb frame, the x_i axis coincident with the first revolute axis and the y_i axis parallel to the base plane. In the initial configuration shown in Fig. 1, the limb twist system is given by

$$\begin{aligned}
\$_{i1}&=(1\ \ 0\ \ 0;0\ \ 0\ \ 0)\\
\$_{i2}&=(1\ \ 0\ \ 0;0\ \ b_2\ \ c_2)\\
\$_{i3}&=(1\ \ 0\ \ 0;0\ \ b_3\ \ c_3).\\
\$_{i4}&=(0\ \ m_4\ \ n_4;a_4\ \ b_4\ \ c_4)\\
\$_{i5}&=(0\ \ m_4\ \ n_4;a_5\ \ b_5\ \ c_5)
\end{aligned} \tag{7}$$

Calculating the reciprocal screws to Eq. (7), we have the limb constraint system

$$\$_{i1}^r=(0\ \ 0\ \ 0;0\ \ n_4\ \ -m_4), \tag{8}$$

which is a couple perpendicular to both $\$_{i3}$ and $\$_{i4}$, i.e., the limb constraint couple is along the common normal of the third revolute axis $\$_{i3}$ and the fourth revolute axis $\$_{i4}$.

Obviously, the three limb constraint couples $\$_{11}^r$, $\$_{21}^r$, and $\$_{31}^r$ are non-coplanar in space. Thus, they are linearly independent and form a 3-system, that is, the mechanism constraint system is a 3-system. The standard base of the mechanism constraint system is given by

$$\begin{aligned}
\$_{m1}^r&=(0\ \ 0\ \ 0;1\ \ 0\ \ 0)\\
\$_{m2}^r&=(0\ \ 0\ \ 0;0\ \ 1\ \ 0),\\
\$_{m3}^r&=(0\ \ 0\ \ 0;0\ \ 0\ \ 1)
\end{aligned} \tag{9}$$

which constrains three rotational DOF of the moving platform. Thus, in the initial configuration, the mechanism has three translational DOF.

Note that in each limb, the first revolute axis $\$_{i1}$ is actually fixed on the base and the fifth revolute axis $\$_{i5}$ is actually fixed on the moving platform. Consequently, after the moving platform undergoes any finite translation, the 3R parallel subchain is always parallel to the base plane and the 2R parallel subchain always bevel the moving platform as well as the base plane. Under

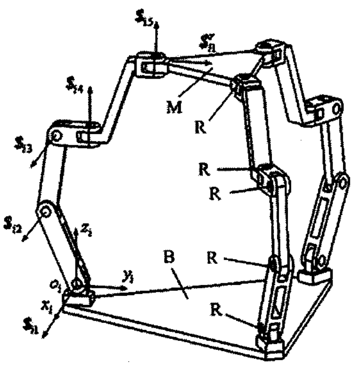

Fig. 2 3-$^{b}\underline{RRR}^{z}\overline{RR}$ parallel mechanism

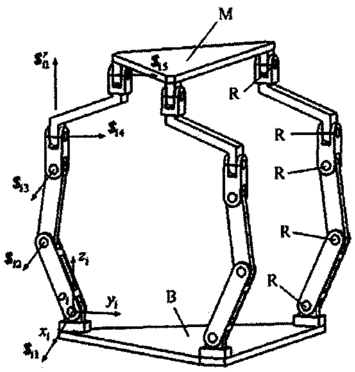

Fig. 3 3-$^{b}\underline{RRR}^{b}\overline{RR}$ parallel mechanism

such a structural condition, the three limb constraint couples are always non-coplanar, which guarantees that the mechanism constraint system remains unchanged. Hence, such a 3-$^{b}\underline{RRR}\overline{RR}$ parallel mechanism has three finite translational DOF.

Because the three limb constraint couples are not coaxial and form a 3-system, we have $\lambda=0$ and $\nu=0$. Using Eq. (3), we have $d=6$. Using Eq. (2), with $n=14$, $g=15$, we have

$$M=6(14-15-1)+15=3. \tag{10}$$

Using equation (6), we also have

$$M=3+0+0=3. \tag{11}$$

2.2 A 4-DOF 3-5R Parallel Mechanism. As shown in Fig. 2, in each limb, the first 3R parallel subchain is parallel to the base plane while the following 2R parallel subchain is perpendicular to the base plane. The three limbs are set symmetrically. We denoted such a mechanism as 3-$^{b}\underline{RRR}^{z}\overline{RR}$, where the superscript z denotes that the last two revolute axes are perpendicular to the base plane.

The limb frame is established the same as that in section 2.1. In the initial configuration shown in Fig. 2, the limb twist system is given by

$$\begin{aligned}
\$_{i1}&=(1\ \ 0\ \ 0;0\ \ 0\ \ 0)\\
\$_{i2}&=(1\ \ 0\ \ 0;0\ \ b_2\ \ c_2)\\
\$_{i3}&=(1\ \ 0\ \ 0;0\ \ b_3\ \ c_3).\\
\$_{i4}&=(0\ \ 0\ \ 1;a_4\ \ b_4\ \ 0)\\
\$_{i5}&=(0\ \ 0\ \ 1;a_5\ \ b_5\ \ 0)
\end{aligned} \tag{12}$$

Calculating the reciprocal screws to Eq. (12), we have the limb constraint system

$$\$_{i1}^{r}=(0\ \ 0\ \ 0;0\ \ 1\ \ 0), \tag{13}$$

which is a couple in the y_i axis, namely, perpendicular to both $\$_{i3}$ and $\$_{i4}$.

Obviously, the three limb constraint couples $\$_{11}^{r}$, $\$_{21}^{r}$ and $\$_{31}^{r}$ are coplanar in space and are parallel to the base plane. Thus, they are linearly dependent and only form a 2-system, that is, the mechanism constraint system is a 2-system. The standard base of the mechanism constraint system is given by

$$\begin{aligned}
\$_{m1}^{r}&=(0\ \ 0\ \ 0;1\ \ 0\ \ 0)\\
\$_{m2}^{r}&=(0\ \ 0\ \ 0;0\ \ 1\ \ 0),
\end{aligned} \tag{14}$$

which constrains two rotational DOF of the moving platform in the base plane. Thus, in the initial configuration, the mechanism has three translational DOF and one rotational DOF about the normal of the base plane.

Note that the feasible motion of the moving platform only includes three translations and one rotation about the normal of the base plane. It can be found without difficulty that after the moving platform undergoes any feasible finite motion, the moving platform is always parallel to the base. Moreover, in each limb, the first revolute axis $\$_{i1}$ is actually fixed on the base and the fifth revolute axis $\$_{i5}$ is actually fixed on the moving platform. Therefore, $\$_{i3}$ is always parallel to the base plane and $\$_{i5}$ is always perpendicular to the base plane. Hence, the limb constraint couple $\$_{i1}^{r}$ is always parallel to the base plane. Considering that the first revolute axis in each limb is not parallel to one another, the three limb constraint couples are always coplanar and form the mechanism constraint system given by Eq. (14). Hence, such a 3-$^{b}\underline{RRR}^{z}\overline{RR}$ is not instantaneous and has three finite translational DOF and one finite rotational DOF about the normal of the base plane.

Because the three limb constraint couples are not coaxial and form a 2-system, we have $\lambda=0$ and $\nu=1$. Using Eq. (3), we have $d=6$. Using Eq. (2), we have

$$M=6(14-15-1)+15+1=4. \tag{15}$$

Using Eq. (6), we also have

$$M=3+0+1=4. \tag{16}$$

2.3 A 5-DOF 3-5R Parallel Mechanism. As shown in Fig. 3, in each limb, the first 3R parallel subchain is parallel to the base plane while the following 2R parallel subchain is also parallel to the base plane and perpendicular to the axes of the first 3R parallel subchain. The three revolute axes adjacent to the base in the three limbs are parallel to each other and the three revolute axes adjacent to the moving platform in the three limbs are parallel to each other. Obviously, under such a geometrical condition, the plane formed by $\$_{i3}$ and $\$_{i4}$ is parallel to the base plane. We denoted such a mechanism as 3-$^{b}\underline{RRR}^{b}\overline{RR}$.

The limb frame is established the same as that in section 2.1. In the initial configuration shown in Fig. 3, the limb twist system is given by

$$\begin{aligned}
\$_{i1}&=(1\ \ 0\ \ 0;0\ \ 0\ \ 0)\\
\$_{i2}&=(1\ \ 0\ \ 0;0\ \ b_2\ \ c_2)\\
\$_{i3}&=(1\ \ 0\ \ 0;0\ \ b_3\ \ c_3).\\
\$_{i4}&=(0\ \ 1\ \ 0;a_4\ \ 0\ \ c_4)\\
\$_{i5}&=(0\ \ 1\ \ 0;a_5\ \ 0\ \ c_5)
\end{aligned} \tag{17}$$

By calculating the screws reciprocal to Eq. (17), we have

$$\$_{i1}^{r}=(0\ \ 0\ \ 0;0\ \ 0\ \ 1). \tag{18}$$

Equation (18) shows that such a single $^{b}\underline{RRR}^{b}\overline{RR}$ limb exerts a constraint couple on the moving platform. The couple is perpendicular to both $\$_{i3}$ and $\$_{i4}$. Hence, all the three limb constraint couples are parallel, thereby being linearly dependent and forming a 1-system. Consequently, the mechanism constraint system is given by

$$\$_{m1}^{r}=(0\ \ 0\ \ 0;0\ \ 0\ \ 1), \tag{19}$$

which constrains the rotational DOF of the moving platform about the normal of the base plane. Hence, in such an initial configuration, the moving platform has three translational DOF and two rotational DOF in the base plane.

Note after any finite translation or rotation about the y_i axis, the limb twist system remains unchanged and the plane formed by $\$_{i3}$ and $\$_{i4}$ is always parallel to the base plane. The mechanism constraint system is the same as Eq. (19).

After any finite rotation about the x_i axis, the limb twist system becomes

$$\begin{aligned}\$_{i1}&=(1\ \ 0\ \ 0;\ \ 0\ \ 0\ \ 0)\\ \$_{i2}&=(1\ \ 0\ \ 0;\ \ 0\ \ b_2\ \ c_2)\\ \$_{i3}&=(1\ \ 0\ \ 0;\ \ 0\ \ b_3\ \ c_3).\\ \$_{i4}&=(0\ \ m_4\ \ n_4;\ \ a_4\ \ b_4\ \ c_4)\\ \$_{i5}&=(0\ \ m_4\ \ n_4;\ \ a_5\ \ b_5\ \ c_5)\end{aligned} \tag{20}$$

By calculating the screws reciprocal to Eq. (20), we have

$$\$_{i1}^{r}=(0\ \ 0\ \ 0;\ \ 0-n_4\ \ m_4). \tag{21}$$

$\$_{i1}^{r}$ in Eq. (21) denotes a constraint couple perpendicular to the plane formed by $\$_{i3}$ and $\$_{i4}$. Since the planes formed by $\$_{i3}$ and $\$_{i4}$ in all the three limbs are parallel to one another, the three limb constraint couples are parallel and equal one couple, that is, they form a 1-system and constrain the rotational DOF of the moving platform about the normal of the plane formed by $\$_{i3}$ and $\$_{i4}$.

In brief, the mechanism constraint system of such a 3-$^{b}\underline{RRR}^{b}\overline{RR}$ parallel mechanism only contains one constraint couple along the normal of the plane formed by $\$_{i3}$ and $\$_{i4}$. Thus, the mechanism loses the rotational DOF about the normal of the plane formed by $\$_{i3}$ and $\$_{i4}$ and has three translational DOF and two rotational DOF. From above analysis, it can be seen that the mechanism constraint system remains unchanged after any finite motion. Hence, the mechanism is not instantaneous.

Because the three limb constraint couples are coaxial, they form a common constraint, we have $\lambda=1$ and $\nu=0$. Using Eq. (3), we have $d=5$. Using equation (3), we have

$$M=5(14-15-1)+15=5. \tag{22}$$

Using Eq. (6), we also have

$$M=3+2\cdot 1+0=5. \tag{23}$$

It is worth mentioning that the reason for the 3-5R parallel mechanism to have four DOF or five DOF is that the part of connecting chains span a planar motion group due to the special geometrical arrangement.

3 Conclusions

It has been shown in this paper that the 3-5R parallel mechanism has different mobility under different structural conditions. When the axes of the second 2R parallel subchain bevel the base plane, the mechanism has three translational DOF; when the axes of the second 2R parallel subchain are perpendicular to the base plane, the mechanism has three translational DOF and one rotational DOF about the normal of the base plane; when the axes of the second 2R parallel subchain in different limb are parallel to one another and to the base plane, and the axes of the first 3R parallel subchain in different limb are parallel to one another, the mechanism has three translational DOF and two rotational DOF. It is also shown that the three 3-5R parallel mechanism can be modularized easily due to the simple structure of the subchains, which can reduce the cost of design and manufacturing.

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