Type Synthesis of 3R2T 5-DOF Parallel Mechanisms Using the Lie Group of Displacements

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***Abstract*—The use of lower mobility parallel manipulators with less than six degrees of freedom (DOFs) has drawn a lot of interest in the area of parallel robots. In this paper, the type synthesis of 3R2T 5-DOF parallel mechanisms (PMs) is performed systematically using the Lie group of displacements, where R denotes a rotational DOF, and T denotes a translational DOF. First, some necessary theoretical fundamentals about the displacement group are recalled. Then, a general approach is proposed for the type synthesis of 3R2T 5-DOF PMs. The limb kinematic chains, which produce the desired displacement manifolds, are synthesized and enumerated. Structural conditions, which guarantee that the intersection of the displacement manifolds generated by the limb is the desired 5-DOF manifold, are presented. An exhaustive enumeration of 3R2T 5-DOF symmetrical PMs is obtained. Finally, an input selection method is proposed.**

***Index Terms*—Displacement Lie group, parallel mechanism (PM), type synthesis.**

I. Introduction

LOWER MOBILITY parallel mechanisms (PMs) are suitable for many tasks requiring less than six degrees of freedom (DOFs). For instance, the 6-DOF PMs currently used in machine tools have a superfluous complexity, since only five DOFs are needed for the tool control. Obviously, using 5-DOF PMs to achieve such tasks can save a lot of cost. As a matter of fact, the machine architecture will be simpler, and the control system and the manufacturing process will be reduced.

According to the discrimination in mobility kind, that is, rotational or translational, the lower mobility PMs can be classified. When referring to mobility, R denotes a rotational DOF, and T denotes a translational DOF. The 5-DOF PMs are sorted in two categories, namely 3R2T and 2R3T. One category 3R2T has three rotational DOFs and two translational DOFs, such as the 3-RRR(RR) PM [1]–[5], where R denotes revolute pair, RRR denotes three successive parallel revolute pairs, and (RR) denotes a 2R spherical subchain; the other category 2R3T has two rotational DOFs and three translational DOFs, such as the 3-RR(RR)R [4]. Particularly, some experts were thinking that no 5-DOF symmetrical PMs can be discovered [6], [7].

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Similarly, the 4-DOF PMs are sorted in three categories, namely, 3R1T [3], [4], [8], 1R3T [9], and 2R2T. The 3-DOF PMs are sorted in four categories, namely, 3T [4], [10]–[13], 3R [14], [15], 2R1T [6], and 1R2T. One interesting example of 2R1T is the 3-revolute-prismatic-spherical (RPS) pyramid PM [16]. This mechanism can perform three independent pure rotations, while the three rotational axes do not intersect at a common point and must be in a hyperbolic paraboloid [17].

This paper will emphasize the 3R2T 5-DOF PM, which produces planar motion together with orientational motion. In a potential application, a 6-DOF hybrid machine can be designed by adding a 1-DOF translational actuated joint. Then the resulting hybrid machine can be used in assembly automation to insert objects into a flat part, which has been fixed parallel to the planar motion. It is worth noticing that the PM can self-comply laterally as well as angularly, reducing insertion force and preventing jams. The PM can work like a remote center compliance device.

Generally, the methods used for the type synthesis of lower-mobility PMs can be divided into two main approaches: *Lie subgroup synthesis method* [18]–[20], which is based on the algebraic properties of a Lie group of the Euclidean displacement set, and *constraint-synthesis method* [2], [21], which is based on the more classical screw theory.

In 1978, Hervé proposed a classification of kinematic chains based on the group algebraic structure of the displacement set, which laid down the foundation of using group theory in mobility analysis, as well as in type synthesis [18]. Hervé also studied the type synthesis of 3-DOF translational PM by Lie group theory [19]. Karouia and Hervé [20] investigated the type synthesis of 3-DOF orientational PMs. Nevertheless, the displacement set $\{\boldsymbol{D}\}$ has also more special properties, which are not reflected only by its Lie group algebraic structure. Consequently, the Lie subgroup synthesis method can not account for very special cases of mobility, an example of which is the paradoxical Bennett mechanism.

The key idea behind the constraint-synthesis method lies in the fact that for a lower mobility PM, the combined effect of all the limb structural constraints determines which DOFs of the moving platform are constrained. This more classical approach can provide successfully many new results. For example, Huang and Li proposed such a constraint-synthesis method for the type synthesis of lower mobility PMs [2] and synthesized some 3-, 4-, and 5-DOF symmetrical parallel manipulators [1], [3]–[5]. Kong and Gosselin studied the type synthesis of 3-DOF spherical parallel manipulators based on screw theory [21].

Besides the two methods mentioned above, Jin *et al.* [22] proposed a synthesis method based on the single-opened-chain

TABLE I
LOWER PAIRS AND LIE SUBGROUPS OF DISPLACEMENTS

Group of displacements	Dimension	Associated kinematic pairs
$\{\boldsymbol{E}\}$	0	Rigid connection
$\{\boldsymbol{R}(N, \boldsymbol{u})\}$	1	Revolute pair R
$\{\boldsymbol{T}(\boldsymbol{v})\}$	1	Prismatic pair P
$\{\boldsymbol{H}(A, p, \boldsymbol{u})\}$	1	Screw pair H
$\{\boldsymbol{C}(N, \boldsymbol{v})\}$	2	Cylindrical pair C
$\{\boldsymbol{G}(\boldsymbol{u})\}$	3	Planar pair G
$\{\boldsymbol{S}(N)\}$	3	Spherical pair S

units. Gao [23] investigated the type synthesis of parallel manipulators using composite joints.

However, because twist and wrench are instantaneous, the last indispensable step of the constraint-synthesis method is to identify whether the synthesized mechanism is only instantaneous or not, which sometimes is difficult. Nevertheless, when using the Lie subgroup synthesis method, this step is not necessary, because it deals directly with finite motion. Furthermore, up until today, the Lie subgroup synthesis method has not been applied for the type synthesis of 4- and 5-DOF PMs.

In this paper, an exhaustive enumeration of 3R2T 5-DOF symmetrical PMs is obtained by the Lie subgroup approach, which not only testifies to our previous work [1]–[5], but also covers 14 novel architectures. Finally, an input selection method is also proposed.

II. BASIC CONCEPTS

It can be readily proven that the set of relative motion allowed by a lower pair constitutes a displacement Lie subgroup [18]. For example, the set of relative motions allowed by a revolute pair forms the Lie subgroup denoted $\{\boldsymbol{R}(N, \boldsymbol{u})\}$ of rotations about a fixed axis, which is designated by $(N, \boldsymbol{u})$. This axis passes through a given point N, and $\boldsymbol{u}$ is the unit vector parallel to the axis. We will say that the revolute pair is a mechanical generator of $\{\boldsymbol{R}(N, \boldsymbol{u})\}$. The Lie subgroups of the displacement Lie group $\{\boldsymbol{D}\}$ that are associated with lower kinematic pairs, are recalled in Table I.

When analyzing a serial kinematic chain composed of rigid bodies $1, 2, \ldots, i-1, i$, the allowed displacements of body i relative to body 1 is a subset of the group of rigid-body motions or displacements. This subset is the composition by implementation of the group product, of all the subgroups associated with the lower kinematic pairs in the kinematic chain. This subset may be a subgroup or only a manifold included in $\{\boldsymbol{D}\}$. For example, consider a moving body connected to a base by a RPS kinematic chain. The displacement of the moving body relative to the base is a subset of the displacement group, which is the five-dimensional (5-D) manifold $\{\boldsymbol{R}(A, \boldsymbol{u})\} \cdot \{\boldsymbol{T}(\boldsymbol{w})\} \cdot \{\boldsymbol{S}(N)\}$. This subset is called a kinematic bond. The RPS kinematic chain is called the mechanical generator of the bond $\{\boldsymbol{R}(A, \boldsymbol{u})\} \cdot \{\boldsymbol{T}(\boldsymbol{w})\} \cdot \{\boldsymbol{S}(N)\}$.

When analyzing a PM, the set of the allowed displacements of the moving platform is the intersection of the displacement sets that are generated by the limbs.

III. TYPE SYNTHESIS OF 3R2T 5-DOF PMs

For convenience, P_{vw} denotes the plane determined by the two one-directional translations, which are characterized by the unit vectors $\boldsymbol{v}$ and $\boldsymbol{w}$.

A. General Approach for the Type Synthesis

The general approach for the type synthesis of 3R2T 5-DOF PMs is as follows.

- Step 1) Describe the desired displacement subset of the moving platform in the 5-DOF PMs by means of a 5-D manifold.
- Step 2) Use the displacement subset found in Step 1 for finding the possible limb kinematic bond and the structural conditions, which guarantee that the intersection of all the limb kinematic bonds is the desired one, given in Step 1.
- Step 3) Find the mechanical generators of the limb kinematic bond found in Step 2.
- Step 4) Construct the 5-DOF PM using the mechanical generators found in Step 3 while following the geometrical condition obtained in Step 2.

Obviously, the key of the whole approach is Step 2, which will be addressed in detail in Sections III-B–G.

B. Kinematic Bond Between the Base and the Moving Platform

Obviously, the three orientational DOFs can be represented by a 3-D displacement subgroup of spherical rotations, namely, $\{\boldsymbol{S}(N)\}$, where N is the rotation center; the two translational DOFs can be represented by a 2-D displacement subgroup of planar translations, namely, $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$. Hence, in such a 3R2T 5-DOF symmetrical PM, the displacement set of the moving platform relative to the base is the composition of the two displacement subgroups, namely, $\{\boldsymbol{S}(N)\} \cdot \{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ or $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\} \cdot \{\boldsymbol{S}(N)\}$, which is termed the kinematic bond between the base and the moving platform. In what follows, we only focus on $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\} \cdot \{\boldsymbol{S}(N)\}$, because the mechanisms obtained from $\{\boldsymbol{S}(N)\} \cdot \{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ are the kinematic inversion of those obtained from $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\} \cdot \{\boldsymbol{S}(N)\}$.

C. Two Expressions of the Bond $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\} \cdot \{\boldsymbol{S}(N)\}$

Because of the closure of products in the subgroup $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$, we have

$$\{\boldsymbol{T}(\boldsymbol{P}_{vw})\} = \{\boldsymbol{T}(\boldsymbol{v})\} \cdot \{\boldsymbol{T}(\boldsymbol{w})\} \tag{1}$$

where $\boldsymbol{v}$ and $\boldsymbol{w}$ are two linearly independent unit vectors, and therefore, form a plane $\boldsymbol{P}_{vw}$. Equation (1) shows that the mechanical generator of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ can be two successive prismatic pairs in direction $\boldsymbol{v}$ and $\boldsymbol{w}$, respectively, denoted as $^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}$, where the superscript denotes the direction of the prismatic pair, and here, the serial order of prismatic pairs does not matter, $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ being a commutative subgroup.

And for the same reason, we have

$$\{\boldsymbol{S}(N)\} = \{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{k})\} \quad (2)$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are three arbitrary linearly independent unit vectors. Obviously, (2) shows that a generator of $\{\boldsymbol{S}(N)\}$ can be a spherical pair of center N or three successive revolute pairs, whose axes intersect at a common point N, denoted by $(^{\mathrm{i}}\mathrm{R}^{\mathrm{j}}\mathrm{R}^{\mathrm{k}}\mathrm{R})_{\mathrm{N}}$.

Considering the product closure in the subgroup $\{\boldsymbol{R}(N,\boldsymbol{u})\}$, we have

$$\{\boldsymbol{R}(N,\boldsymbol{u})\} = \{\boldsymbol{R}(N,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}. \quad (3)$$

Combining (1)–(3), we have

$$\begin{aligned}\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\} &= \{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\\ &= \{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}. \quad (4)\end{aligned}$$

Because the unit vector $\boldsymbol{u}$ can be set perpendicular to the plane $\boldsymbol{P}_{vw}$, the composition of $\{\boldsymbol{T}(\boldsymbol{v})\}$, $\{\boldsymbol{T}(\boldsymbol{w})\}$, and $\{\boldsymbol{R}(N,\boldsymbol{u})\}$ produces the subgroup of planar gliding displacements $\{\boldsymbol{G}(\boldsymbol{u})\}$, the plane being perpendicular to the unit vector $\boldsymbol{u}$, that is

$$\{\boldsymbol{G}(\boldsymbol{u})\} = \{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\} \quad (5)$$

which corresponds to the mechanical generator $^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}^{\mathrm{u}}\mathrm{R}$.

Substituting (5) into (4) leads to

$$\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\} = \{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}. \quad (6)$$

Equation (6) shows that the kinematic bond $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ is equivalent to the kinematic bond $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ kinematically.

Because of the product closure in the subgroup $\{\boldsymbol{G}(\boldsymbol{u})\}$, (5) can also be written as

$$\begin{aligned}\{\boldsymbol{G}(\boldsymbol{u})\} &= \{\boldsymbol{R}(N,\boldsymbol{u})\}\cdot\{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\\ &= \{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\} \quad (7)\end{aligned}$$

which corresponds to the mechanical generators $^{\mathrm{u}}\mathrm{R}^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}$ and $^{\mathrm{v}}\mathrm{P}^{\mathrm{u}}\mathrm{R}^{\mathrm{w}}\mathrm{P}$. For the same reason, we can list all mechanical generators of $\{\boldsymbol{G}(\boldsymbol{u})\}$, which are shown in Table II.

D. Limb Kinematic Bonds

Let $\{\boldsymbol{L}_i\}$ denote the kinematic bond produced by the ith limb. It is worth remarking that $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ must be the intersection of all the limb kinematic bonds produced by all limb kinematic chains. Hence, we can write

$$\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\} = \bigcap_{i=1}^{n}\{\boldsymbol{L}_i\} \quad (8)$$

or

$$\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} = \bigcap_{i=1}^{n}\{\boldsymbol{L}_i\}. \quad (9)$$

Because one can readily prove

$$\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}\cap\{\boldsymbol{T}(\boldsymbol{P}'_{vw})\}\cdot\{\boldsymbol{S}(N')\} = \{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\} \quad (10)$$

TABLE II
MECHANICAL GENERATORS OF $\{\boldsymbol{G}(\boldsymbol{u})\}$

	Kinematic bond	Mechanical generator
$\{\boldsymbol{G}(\boldsymbol{u})\}$	$\{\boldsymbol{T}(\boldsymbol{v})\}.\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{R}(N,\boldsymbol{u})\}$	$^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}^{\mathrm{u}}\mathrm{R}$
	$\{\boldsymbol{R}(N,\boldsymbol{u})\}.\{\{\boldsymbol{T}(\boldsymbol{v})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	$^{\mathrm{u}}\mathrm{R}^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}$
	$\{\boldsymbol{T}(\boldsymbol{v})\}.\{\boldsymbol{R}(N,\boldsymbol{u})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	$^{\mathrm{v}}\mathrm{P}^{\mathrm{u}}\mathrm{R}^{\mathrm{w}}\mathrm{P}$
	$\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{R}(B,\boldsymbol{u})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	$^{\mathrm{u}}\mathrm{R}^{\mathrm{u}}\mathrm{R}^{\mathrm{w}}\mathrm{P}$
	$\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{R}(B,\boldsymbol{u})\}$	$^{\mathrm{u}}\mathrm{R}^{\mathrm{w}}\mathrm{P}^{\mathrm{u}}\mathrm{R}$
	$\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{R}(B,\boldsymbol{u})\}$	$^{\mathrm{w}}\mathrm{P}^{\mathrm{u}}\mathrm{R}^{\mathrm{u}}\mathrm{R}$
	$\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{R}(B,\boldsymbol{u})\}.\{\boldsymbol{R}(C,\boldsymbol{u})\}$	$^{\mathrm{u}}\mathrm{R}^{\mathrm{u}}\mathrm{R}^{\mathrm{u}}\mathrm{R}$

if and only if $\boldsymbol{P}_{vw}//\boldsymbol{P}'_{vw}$ and $N' = N$. Similarly, we also have

$$\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}\cap\{\boldsymbol{G}(\boldsymbol{u}')\}\cdot\{\boldsymbol{S}(N')\} = \{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} \quad (11)$$

if and only if $\boldsymbol{u} = \boldsymbol{u}'$ and $N' = N$.

Thus, (8) and (9) shows that each kinematic bond $\{\boldsymbol{L}_i\}$ must contain a subgroup of rotations $\{\boldsymbol{S}(N_i)\}$, and the rotation centers N_i must coincide at N in order to guarantee that the bond intersection still contains a subgroup of spherical rotations. For analogous reasons, the planes that are determined by $\boldsymbol{P}_{vw}$ or by $\boldsymbol{u}$ must be parallel. Hence, the orientational centers of the mechanical generators of $\{\boldsymbol{S}(N)\}$ must coincide. If the use of double spherical joints is prohibited, the PM can not implement spherical pairs.

Based on above analysis, we can take generators of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ or $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ as the limb kinematic chains. In the synthesis of structurally asymmetrical PM, only one limb that generates $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ and other limbs that generate $\{\boldsymbol{D}\}$ can also be chosen. The expression $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ includes a subgroup $\{\boldsymbol{R}(N,\boldsymbol{u})\}$ of rotations, which play no role in the bond. As a matter of fact, the intersection of $\{\boldsymbol{G}(\boldsymbol{u})\}$ and $\{\boldsymbol{S}(N)\}$ is $\{\boldsymbol{R}(N,\boldsymbol{u})\}$, $\{\boldsymbol{G}(\boldsymbol{u})\}\cap\{\boldsymbol{S}(N)\} = \{\boldsymbol{R}(N,\boldsymbol{u})\}$, whereas $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cap\{\boldsymbol{S}(N)\} = \{E\}$.

E. Mechanical Generators of Limb Kinematic Bonds

The limb kinematic chains can be obtained simply by the serial arrangement of the mechanical generators of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ and $\{\boldsymbol{S}(N)\}$, or those of $\{\boldsymbol{G}(\boldsymbol{u})\}$ and $\{\boldsymbol{S}(N)\}$.

1) Mechanical Generators of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$*:* The mechanical generator of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}$ is a sequence of two nonparallel prismatic pairs, which lie in the plane $\boldsymbol{P}_{vw}$, namely, $^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}$. In addition, $\{\boldsymbol{S}(N)\}$ has two mechanical generators, a spherical pair S_{N}, or a 3R spherical subchain $(^{\mathrm{i}}\mathrm{R}^{\mathrm{j}}\mathrm{R}^{\mathrm{k}}\mathrm{R})_{\mathrm{N}}$. Hence, the mechanical generator of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ is $^{\mathrm{v}}\mathrm{P}^{\mathrm{w}}\mathrm{P}(^{\mathrm{i}}\mathrm{R}^{\mathrm{j}}\mathrm{R}^{\mathrm{k}}\mathrm{R})_{\mathrm{N}}$.

2) Mechanical Generators of $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$*:* Any sequence of a mechanical generator of $\{\boldsymbol{G}(\boldsymbol{u})\}$ and a mechanical generator of $\{\boldsymbol{S}(N)\}$ can make up a limb. However, each of these limbs has a passive 1-DOF mobility.

Mechanical generators of $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ with no passive mobility can be obtained by elimination of one DOF in $\{\boldsymbol{G}(\boldsymbol{u})\}$, or by elimination of one DOF in $\{\boldsymbol{S}(N)\}$.

TABLE III
MECHANICAL GENERATORS OF $\{\boldsymbol{G}2(\boldsymbol{u})\}$

	Kinematic bond	Mechanical generator
$\{\boldsymbol{G}2(\boldsymbol{u})\}$	$\{\boldsymbol{T}(\boldsymbol{v})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	${}^{v}P{}^{w}P$
	$\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	${}^{u}R{}^{w}P$
	$\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{R}(A,\boldsymbol{u})\}$	${}^{w}P{}^{u}R$
	$\{\boldsymbol{R}(A,\boldsymbol{u})\}.\{\boldsymbol{R}(B,\boldsymbol{u})\}$	${}^{u}R{}^{u}R$

Expressions of $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ with no superfluous parameter can be obtained by elimination of the redundancy of the 1-D subgroup $\{\boldsymbol{R}(N,\boldsymbol{u})\}$ in $\{\boldsymbol{G}(\boldsymbol{u})\}$, or by its elimination in $\{\boldsymbol{S}(N)\}$. By doing this, we obtain two subfamilies of limbs. If we denote $\{\boldsymbol{G}2(\boldsymbol{u})\}$ a 2-D manifold included in $\{\boldsymbol{G}(\boldsymbol{u})\}$, and $\{\boldsymbol{S}2(N)\}$ a 2-D manifold included in $\{\boldsymbol{S}(N)\}$, the two subfamilies are represented by $\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$, and $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}\cdot\{\boldsymbol{S}2(N)\}$ is the product of two distinct rotation subgroups with axes converging at the point N.

- Mechanical generators of $\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$
  Using (3), we can eliminate in $\{\boldsymbol{G}(\boldsymbol{u})\}$ the factor $\{\boldsymbol{R}(N,\boldsymbol{u})\}$, which represents the passive mobility in $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$. For example, we have

$$\begin{aligned}\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} &= \{\boldsymbol{R}(A,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(B,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\\ &= \{\boldsymbol{R}(A,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(B,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{u})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\\ &= \{\boldsymbol{R}(A,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(B,\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}\end{aligned} \quad (12)$$

  where $\{\boldsymbol{R}(A,\boldsymbol{u})\}\cdot\{\boldsymbol{R}(B,\boldsymbol{u})\}$ is a possible case of $\{\boldsymbol{G}2(\boldsymbol{u})\}$ among those, which are listed in Table III.
  The mechanical generators of the type $\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ are obtained by putting in series a mechanical generator of the $\{\boldsymbol{G}2(\boldsymbol{u})\}$ bonds and a 3R spherical subchain. For example, one mechanical generator of $\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ is a ${}^{u}R{}^{u}R({}^{i}R{}^{j}R{}^{k}R)_N$ kinematic chain.
- Mechanical generators of $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}$
  Substituting (3) into $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$ leads to

$$\begin{aligned}\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} &= \{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\\ &= \{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}.\end{aligned} \quad (13)$$

  In (13), $\{\boldsymbol{S}2(N)\}$ can be associated with a 2R spherical subchain, denoted as $({}^{i}R{}^{j}R)_N$. The limb kinematic chains of the type $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}$ are obtained by putting in series one of the mechanical generators of $\{\boldsymbol{G}(\boldsymbol{u})\}$ and a 2R spherical subchain, provided that the unit vectors $\boldsymbol{u}$, $\boldsymbol{i}$, and $\boldsymbol{j}$ are linearly independent. For example, one mechanical generator of $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}$ is a ${}^{u}R{}^{u}R{}^{u}R({}^{i}R{}^{j}R)_N$ limb kinematic chain. Other chains are sequences of other $\{\boldsymbol{G}(\boldsymbol{u})\}$ generators given in Table I, and $({}^{i}R{}^{j}R)_N$.

TABLE IV
MECHANICAL GENERATORS OF $\{\boldsymbol{C}(N,\boldsymbol{w})\}$

	Kinematic bond	Mechanical generator
$\{\boldsymbol{C}(N,\boldsymbol{w})\}$	$\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{R}(N,\boldsymbol{w})\}$	${}^{w}P{}^{w}R$
	$\{\boldsymbol{R}(N,\boldsymbol{w})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	${}^{w}R{}^{w}P$
	$\{\boldsymbol{T}(\boldsymbol{w})\}.\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{p})\}$	${}^{w}P{}^{w}H$
	$\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{p})\}.\{\boldsymbol{T}(\boldsymbol{w})\}$	${}^{w}H{}^{w}P$
	$\{\boldsymbol{R}(N,\boldsymbol{w})\}.\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{p})\}$	${}^{w}R{}^{w}H$
	$\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{p})\}.\{\boldsymbol{R}(N,\boldsymbol{w})\}$	${}^{w}H{}^{w}R$
	$\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{p})\}.\{\boldsymbol{H}(N,\boldsymbol{w},\mathrm{q})\}$	${}^{w}H{}^{w}H$

3) Chains With 2-DOF Joints: 2-DOF kinematic joints, such as cylindrical pairs and universal joints, can also be employed by some appropriate combinations.

- Chains with cylindrical pairs
  A cylindrical pair is a 2-DOF lower kinematic pair and is a mechanical generator of the subgroup of cylindrical displacements $\{\boldsymbol{C}(N,\boldsymbol{w})\}$, the pair axis being characterized by a point N and a unit vector $\boldsymbol{w}$. The mechanical generators of $\{\boldsymbol{C}(N,\boldsymbol{w})\}$ are listed in Table IV. Obviously, none of $\{\boldsymbol{G}2(\boldsymbol{u})\}$, $\{\boldsymbol{G}(\boldsymbol{u})\}$, $\{\boldsymbol{S}2(N)\}$, and $\{\boldsymbol{S}(N)\}$ contains a $\{\boldsymbol{C}(N,\boldsymbol{w})\}$. Nevertheless, the $\{\boldsymbol{C}(N,\boldsymbol{w})\}$ can result from the combination of a last factor in the products that generate $\{\boldsymbol{G}2(\boldsymbol{u})\}$ or $\{\boldsymbol{G}(\boldsymbol{u})\}$, and the first factor in $\{\boldsymbol{S}(N)\}$ or $\{\boldsymbol{S}2(N)\}$. Moreover, the last factor in $\{\boldsymbol{G}2(\boldsymbol{u})\}$ or $\{\boldsymbol{G}(\boldsymbol{u})\}$ decomposed in subgroup products must be a 1-D translational subgroup $\{\boldsymbol{T}(\boldsymbol{h})\}$, where $\boldsymbol{h}$ can be $\boldsymbol{v}$ or $\boldsymbol{w}$, or the linear combination of both.
  Let us first introduce cylindrical joints in limbs of the family $\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$. Only two kinematic bonds of the kind $\{\boldsymbol{G}2(\boldsymbol{u})\}$ can be used to generate the cylindrical subgroup. They are $\{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}$ and $\{\boldsymbol{R}(A,\boldsymbol{u})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}$.
  In (4), the unit vector $\boldsymbol{u}$ can be $\boldsymbol{w}$ provided that $\boldsymbol{w}$, $\boldsymbol{i}$, and $\boldsymbol{j}$ are linearly independent. The composition of $\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{w})\}$ is a subgroup of cylindrical displacements $\{\boldsymbol{C}(N,\boldsymbol{w})\}$. Hence

$$\begin{aligned}\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\} &= \{\boldsymbol{T}(\boldsymbol{v})\}\cdot\{\boldsymbol{C}(N,\boldsymbol{w})\}\{\boldsymbol{R}(N,\boldsymbol{i})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\end{aligned} \quad (14)$$

  whose mechanical generator is a ${}^{v}P{}^{w}C_N({}^{i}R{}^{j}R)_N$ limb kinematic chain, where the subscript N in ${}^{w}C_N$ denotes that the axis of the cylindrical joint has to pass through the point N. The displacement group is not commutative, and therefore, here ${}^{w}C_N^{v}P(RR)_N$ can not be an adequate limb.
  Similarly, we have

$$\begin{aligned}\{\boldsymbol{G}2(\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} &= \{\boldsymbol{R}(N',\boldsymbol{u})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{i})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{k})\}\\ &= \{\boldsymbol{R}(N',\boldsymbol{u})\}\cdot\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{w})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{k})\}\\ &= \{\boldsymbol{R}(N',\boldsymbol{u})\}\cdot\{\boldsymbol{C}(N,\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{j})\}\\ &\quad\cdot\{\boldsymbol{R}(N,\boldsymbol{k})\}\end{aligned} \quad (15)$$

TABLE V
MECHANICAL GENERATORS WITH CYLINDRICAL JOINT

$\{\boldsymbol{G}2(\boldsymbol{u})\}.\{\boldsymbol{S}(N)\}$	$\{\boldsymbol{G}(\boldsymbol{u})\}.\{\boldsymbol{S}2(N)\}$
${}^{v}P{}^{w}C_N({}^{j}R{}^{k}R)_N$	${}^{u}R{}^{v}P{}^{w}C_N{}^{k}R_N$
${}^{u}R{}^{w}C_N({}^{j}R{}^{k}R)_N$	${}^{u}R{}^{u}R{}^{w}C_N{}^{k}R_N$
	${}^{v}P{}^{u}R{}^{w}C_N{}^{k}R_N$

TABLE VI
SPECIAL SUBFAMILY OF MECHANICAL GENERATORS

$\{\boldsymbol{G}2(\boldsymbol{u})\}.\{\boldsymbol{S}(N)\}$	$\{\boldsymbol{G}(\boldsymbol{u})\}.\{\boldsymbol{S}2(N)\}$
${}^{u}R{}^{w}R_N{}^{w}P({}^{j}R{}^{k}R)_N$	${}^{u}R{}^{v}P{}^{w}R_N{}^{w}P{}^{k}R_N$
${}^{v}P{}^{w}R_N{}^{w}P({}^{j}R{}^{k}R)_N$	${}^{u}R{}^{u}R{}^{w}R_N{}^{w}P{}^{k}R_N$
	${}^{v}P{}^{u}R{}^{w}R_N{}^{w}P{}^{k}R_N$

whose mechanical generator is a ${}^{u}R{}^{w}C_N({}^{j}R{}^{k}R)_N$ limb kinematic chain.

By the same way, we can also look for the mechanical generators with cylindrical pairs from the bond representation $\{\boldsymbol{G}(\boldsymbol{u})\}\cdot\{\boldsymbol{S}2(N)\}$. All of them are listed in Table V. It is worth mentioning that here we neglect the mechanical generators with helical pairs. If helical pairs are taken into consideration, more new architectures can be obtained.

Considering the permitted commutation of factors in the $\{\boldsymbol{T}(\boldsymbol{w})\}\cdot\{\boldsymbol{R}(N,\boldsymbol{w})\}$ bond of cylindrical subgroup $\{\boldsymbol{C}(N,\boldsymbol{w})\}$, a special subfamily (Table VI) can be obtained by simply replacing the ${}^{w}C_N$ in Table V by ${}^{w}P{}^{w}R_N$ or ${}^{w}R_N{}^{w}P$.

- Generation of universal joint

  If in a serial arrangement of two revolute pairs ${}^{u}R$ and ${}^{i}R$, the axes are intersecting at point N with a right angle, then the sequence of two R pairs can be named a universal joint ${}^{ui}U_N$ and the corresponding limbs are listed in Table VII.

F. Enumeration of Mechanisms

Based on the above analysis, the structural condition of limb kinematic chain of 3R2T 5-DOF PMs is obvious. It can be stated as follows.

For an 3R2T 5-DOF PM, the limb kinematic chain must contain a 3R or a 2R spherical subchain; the prismatic pairs must lie in the plane P_{vw}; the revolute axes, except those in the 2R or 3R spherical subchain, must be perpendicular to the plane P_{vw}.

The exhaustive enumeration of the limb kinematic chains of the 3R2T 5-DOF PMs is given in Fig. 1.

Based on (8) and (9), each kinematic bond $\{\boldsymbol{L}_i\}$ has to include the 5-D manifold $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ or its equivalents. One generator of $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$ is enough to produce the desired bond, and other limbs can be generators of the 6-D displacement Lie group $\{\boldsymbol{D}\}$. However, we want the structural type of all limbs to be the same. Then all limbs must be generators of the same 5-D bond $\{\boldsymbol{T}(\boldsymbol{P}_{vw})\}\cdot\{\boldsymbol{S}(N)\}$. This bond is characterized by the plane that is determined by $\boldsymbol{v}$ and w, and by the

TABLE VII
MECHANICAL GENERATORS WITH UNIVERSAL JOINT

$\{\boldsymbol{G}2(\boldsymbol{u})\}.\{\boldsymbol{S}(N)\}$	$\{\boldsymbol{G}(\boldsymbol{u})\}.\{\boldsymbol{S}2(N)\}$
${}^{v}P{}^{ui}U_N({}^{j}R{}^{k}R)_N$	${}^{w}P{}^{v}P{}^{ui}U_N{}^{j}R_N$
${}^{u}R{}^{ui}U_N({}^{j}R{}^{k}R)_N$	${}^{u}R{}^{w}P{}^{ui}U_N{}^{j}R_N$
${}^{uw}U_N{}^{w}P({}^{i}R{}^{j}R)_N$	${}^{v}P{}^{u}R{}^{wi}U_N{}^{k}R_N$
	${}^{u}R{}^{u}R{}^{ui}U_N{}^{j}R_N$
	${}^{u}R{}^{uw}U_N{}^{w}P{}^{k}R_N$
	${}^{v}P{}^{uw}U_N{}^{w}P{}^{k}R_N$

point N. Hence, we obtain the structural condition at the mechanism level.

In each limb, the plane $\boldsymbol{P}_{vw}$ must be parallel to the given unit vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ and the center must be the same point N.

Based on above analysis, we can enumerate the 3R2T 5-DOF symmetrical PMs, which are listed in Table VIII. 14 of the 30 architectures in Table VIII are revealed for the first time, which are marked with a superscript *. The kinematic inversions of these architectures can be considered.

It is pointed out that asymmetrical 3R2T 5-DOF PMs can be obtained using different limbs in Table VIII, while obeying the structural condition of the whole mechanism.

G. Examples

The following examples have three rotational DOF and two translational DOF in the xy plane. The translational DOF along the z axis is constrained. Hence, in what follows, the superscript u is replaced by z, and the unit vector $\boldsymbol{v}$ and $\boldsymbol{w}$ are in the xy plane. Fig. 2 shows a $-3{}^{x}P{}^{y}P{}^{zy}U_N^{k}R_N$ PM, where M denotes the moving platform and B denotes the base. Fig. 3 shows a $3-{}^{z}R{}^{z}R{}^{x}C_N^{j}R_N$ PM.

IV. INPUT SELECTION METHOD

The choice of actuated pairs cannot be arbitrary [21], [22]. Otherwise, one or more DOF of the mechanism may be uncontrollable, and input interference may also happen.

When all the actuated pairs in a PM are locked, the moving platform loses its DOF and cannot undergo any motion. Then, the intersection of all the limb kinematic bonds is $\{\boldsymbol{E}\}$. Following the steps described below, one can perform the input selection.

Step 1) Write out the limb kinematic bond of a given PM.
Step 2) Select the actuated pairs and replace the displacement subgroup associated with the actuated pairs by the identical transformation $\boldsymbol{E}$.
Step 3) Perform the intersection of all the new reduced limb kinematic bonds.
Step 4) If the intersection obtained in Step 3 is $\{\boldsymbol{E}\}$, the input selection is appropriate.

Note that for a lower mobility PM, generally there are several available sets of actuated pairs.

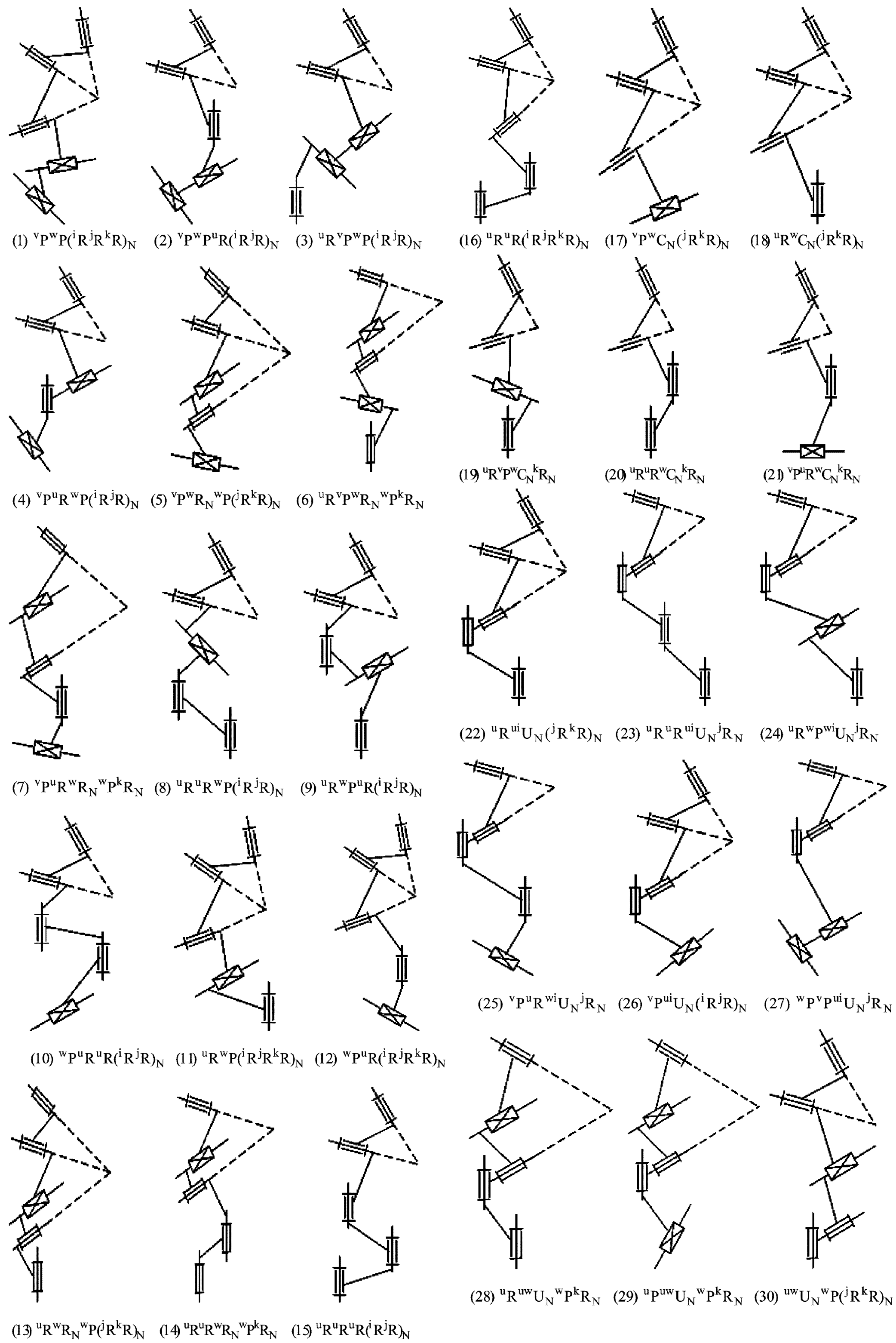

Fig. 1. Limbs of 3R2T 5-DOF PM.

For example, let us consider a $5-{}^{z}R^{z}R(^{i}R^{j}R^{k}R)_N$ PM. Generally, the actuated joints are preferably expected to be on the base. Thus, we attach five actuators on the five grounded revolute pairs. The original ith limb kinematic bond of $5-{}^{z}R^{z}R(^{i}R^{j}R^{k}R)_N$ PM is $\{\boldsymbol{R}_i(A_i,\boldsymbol{u})\}\cdot\{\boldsymbol{R}_i(B_i,\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$. Then, giving the value $\{\boldsymbol{E}\}$ to $\{\boldsymbol{R}_i(A_i,\boldsymbol{u})\}$, which expresses

TABLE VIII
EXHAUSTIVE ENUMERATION OF SYMMETRICAL 3R2T 5-DOF PMS

With two prismatic pair	$m-{}^{v}P{}^{w}P({}^{i}R{}^{j}R{}^{k}R)_N$	$m-{}^{u}R{}^{v}P{}^{w}P({}^{j}R{}^{k}R)_N$	$m-{}^{v}P{}^{u}R{}^{w}P({}^{j}R{}^{k}R)_N{}^{*}$	$m-{}^{v}P{}^{w}P{}^{u}R({}^{i}R{}^{j}R)_N$
	$m-{}^{v}P{}^{w}R_N{}^{w}P({}^{j}R{}^{k}R)_N{}^{*}$	$m-{}^{u}R{}^{v}P{}^{w}R_N{}^{w}P{}^{k}R_N{}^{*}$	$m-{}^{v}P{}^{u}R{}^{w}R_N{}^{w}P{}^{k}R_N{}^{*}$	
With one prismatic pair	$m-{}^{u}R{}^{u}R{}^{w}P({}^{i}R{}^{j}R)_N$	$m-{}^{u}R{}^{w}P{}^{u}R({}^{i}R{}^{j}R)_N$	$m-{}^{w}P{}^{u}R{}^{u}R({}^{i}R{}^{j}R)_N$	$m-{}^{u}R{}^{w}P({}^{i}R{}^{j}R{}^{k}R)_N$
	$m-{}^{w}P{}^{u}R({}^{i}R{}^{j}R{}^{k}R)_N$	$m-{}^{u}R{}^{w}R_N{}^{w}P({}^{i}R{}^{j}R)_N{}^{*}$	$m-{}^{u}R{}^{u}R{}^{w}R_N{}^{w}P{}^{k}R_N{}^{*}$	
With no prismatic pairs	$m-{}^{u}R{}^{u}R({}^{i}R{}^{j}R{}^{k}R)_N$	$m-{}^{u}R{}^{u}R{}^{u}R({}^{i}R{}^{j}R)_N$		
With cylindrical pairs	$m-{}^{v}P{}^{w}C_N({}^{j}R{}^{k}R)_N{}^{*}$	$m-{}^{u}R{}^{w}C_N({}^{j}R{}^{k}R)_N$	$m-{}^{u}R{}^{v}P{}^{w}C_N{}^{k}R_N{}^{*}$	$m-{}^{u}R{}^{u}R{}^{w}C_N{}^{k}R_N{}^{*}$
	$m-{}^{v}P{}^{u}R{}^{w}C_N{}^{k}R_N{}^{*}$			
With universal pairs	$m-{}^{v}P{}^{ui}U_N({}^{j}R{}^{k}R)_N$	$m-{}^{u}R{}^{ui}U_N({}^{j}R{}^{k}R)_N$	$m-{}^{uw}U_N\,{}^{w}P({}^{i}R{}^{j}R)_N{}^{*}$	$m-{}^{w}P{}^{v}P{}^{ui}U_N\,{}^{j}R_N{}^{*}$
	$m-{}^{u}R{}^{w}P{}^{ui}U_N\,{}^{j}R_N$	$m-{}^{v}P{}^{u}R{}^{wi}U_N\,{}^{k}R_N$	$m-{}^{u}R{}^{u}R{}^{ui}U_N\,{}^{j}R_N$	$m-{}^{u}R{}^{uw}U_N\,{}^{w}P{}^{k}R_N{}^{*}$
	$m-{}^{v}P{}^{uw}U_N\,{}^{w}P{}^{k}R_N{}^{*}$			

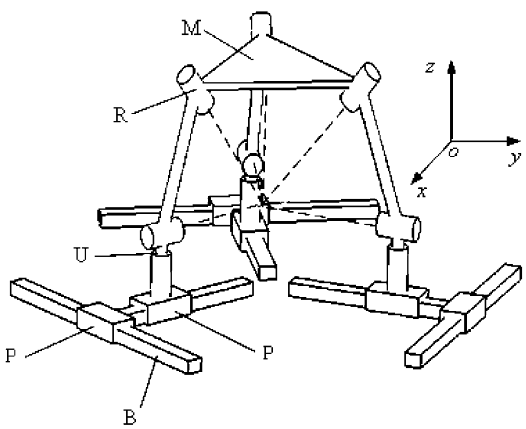

Fig. 2. $3-{}^{x}P{}^{y}P{}^{xy}U^{k}_{N}R_N$ PM.

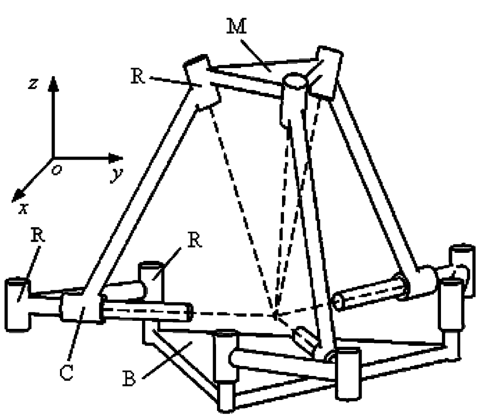

Fig. 3. $3-{}^{z}R{}^{z}R{}^{x}C^{j}_{N}R_N$ PM.

the pair locking, the new limb kinematic bond becomes $\{\boldsymbol{R}_i(B_i,\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\}$. Obviously, we have

$$\{\boldsymbol{S}(N)\} = \bigcap_{i=1}^{5}\{\boldsymbol{R}_i(B_i,\boldsymbol{u})\}\cdot\{\boldsymbol{S}(N)\} \tag{16}$$

which shows that even though the five actuated revolute pairs are locked, the moving platform still has three rotational DOF. Thus, the selection of the five grounded revolute pairs is not adequate.

Following the steps above, we can find the correct set of inputs for this mechanism. Two grounded revolute pairs in any two limbs and one revolute pair in the 3R spherical subchain from each of the other three limbs can be selected as actuated pairs.

Using the input selection method proposed in this section, we can also find that no 3R2T PM with actuation symmetry can be obtained.

V. CONCLUSIONS

1) Properties that take birth from the Lie group algebraic structure of the finite displacement set can be applied to the type synthesis of 3R2T 5-DOF PM, thus providing effective results. It is worth noticing that the general approach proposed in this paper can also be employed for the type synthesis of other lower mobility PMs.
2) An exhaustive enumeration of symmetrical 3R2T 5-DOF PMs is obtained, in which 14 novel architectures are revealed for the first time.
3) The input selection condition is that when all the actuated pairs in a PM are locked, the intersection of all the limb kinematic bonds is $\{\boldsymbol{E}\}$ if the input selection is correct.
4) Different suitable limb kinematic chains, which follow the required structural conditions, can also make up asymmetrical PMs.

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