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Type Synthesis of Symmetrical Lower-mobility Parallel Mechanisms Using the Constraint-synthesis Method

Abstract

The lower-mobility parallel mechanism (PM) has fewer than six degrees of freedom (DoFs) and requires fewer links, joints and actuators than its 6-DoF counterpart. Hence, the use of the lower-mobility PM may save much cost in those applications requiring less mobility. In this paper, the type synthesis of symmetrical lower-mobility PMs is performed systematically using the constraint-synthesis method. First, some important theoretical fundamentals of type synthesis of lower-mobility PMs are investigated, including a classification of symmetry of lower-mobility PMs, the identification of instantaneous mechanisms and input selection method. Moreover, the difficult problem of how to correctly apply the general Grübler-Kutzbach criterion to lower-mobility PMs is solved. A mobility analysis of PMs with a closed loop in the limbs is also presented to demonstrate the validity. The type synthesis of 5-DoF and 4-DoF PMs is then dealt with in detail. Enumerations of novel 5-DoF and 4-DoF PMs are presented. The characteristics of constraint and structure of 5-DoF and 4-DoF PMs are also proposed. In addition, some novel 3-DoF PMs are presented.

KEY WORDS—parallel mechanisms, mobility analysis, design theory, type synthesis

1. Introduction

A parallel mechanism (PM) is one in which two or more serial kinematic chains connect the moving platform to the base. PMs can offer advantages over their serial counterparts in terms of rigidity, dynamic performances and accuracy. In recent years, the research and use of PMs have evolved from the general six-degrees-of-freedom (6-DoF) PMs to the lower-mobility PMs, which have fewer than six DoFs. The root cause is that many applications requiring fewer than six DoFs can be performed by the lower-mobility PM, which is simpler in structure and consequently cheaper than its 6-DoF counterpart. A typical example is that the 6-DoF PMs currently used in machining suffer from their complexity since only five DoFs of the tool need to be controlled.

In what follows, the kinematic pairs will be denoted with the following symbols: P for prismatic pair, R for revolute pair, U for universal joint, C for cylindrical pair, H for helical pair and S for spherical pair.

The 3-DoF PMs have been extensively studied. Hunt (1983) proposed a 3-DoF 3-RPS PM. Clavel (1988) proposed the well-known 3-DoF translational PM, Delta robot. Tsai (1996) proposed the 3-UPU 3-DoF translational PM. Di Gregorio and Parenti-Castelli (1999) studied the 3-UPU translational PM. Di Gregorio (2000) studied the 3-RUU translational PM. Carricato and Parenti-Castelli (2002) investigated the type synthesis and classification of translational PMs and identified special families of fully-isotropic translational PMs for the first time.

Gosselin and his colleagues (1989, 1992, 1993, 1995, 1997) studied the 3-DoF 3-3R spherical PMs systematically and built a fast 3-DoF camera-orienting device. Di Gregorio (2001) proposed a new 3-DoF spherical PM, the 3-URC wrist, which belongs to the family of wrists first introduced by Karouia and Hervé (2000).

In addition, several special 3-DoF PMs have been proposed by Huang and Fang (1996), including a pyramid 3-RPS PM. This mechanism can perform three independent pure rotations while the three rotational axes do not intersect at a common point but lie in a hyperbolic paraboloid (Huang and Wang 2001). They also studied all the possible twist motions with different pitches of such a PM. Tsai (1999) proposed an

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enumeration of a class of 3-DoF PMs based on the structural characteristics of PMs.

As far as 4-DoF PMs are concerned, Pierrot and Company (1999) proposed the 4-DoF H4 robot. Rolland (1999) proposed two 4-DoF PMs, the Manta and the Kanuk, for material handling. They also used parallelograms to eliminate rotations. Zhao and Huang (2000a) proposed a 4-DoF 4-URU PM, which has three translational DoFs and one rotational DoF. Zlatanov and Gosselin (2001) proposed a 4-DoF PM, which has three rotational DoFs and one translational DoF along the normal of the moving platform plane. Huang and Li (2002a) invented two 4-DoF PMs, which have three rotational DoFs and one translational DoF along the normal of the base plane.

As for the 5-DoF PM, however, it was generally believed that no symmetrical 5-DoF PMs existed. Hunt (1978, 1983) believed that the symmetrical 5-DoF PM was instantaneous. There were no symmetrical 5-DoF PMs until Huang and Li (2001) filled this gap.

Researchers have become increasingly interested in the development of an effective method for finding new lower-mobility PMs. Generally, the methods used for the type synthesis of lower-mobility parallel mechanisms can be divided into three approaches: the *Lie subgroup synthesis method*, the *constraint-synthesis method* and the *motion-synthesis method*.

The Lie subgroup synthesis method is based on the algebraic properties of a Lie group of the Euclidean displacement set. Hervé and Sparacino (1991) performed a systematical type synthesis of 3-DoF translational PMs with 4-DoF legs. Hervé (1999) suggested the use of Lie group theory in designing new PMs. Karouia and Hervé (2000, 2002) investigated the type synthesis of 3-DoF orientational PMs.

The constraint-synthesis method is based on screw theory. A 3-RRRH translational PM was synthesized by reciprocal screw theory (Huang, Kong and Fang 1997). A prototype of the constraint-synthesis method was proposed and applied to the type synthesis of 4-DoF PMs (Huang, Zhao and Li 2000). A complete general constraint-synthesis method was proposed by Huang and Li (2002b). Novel symmetrical 3-DoF, 4-DoF and 5-DoF PMs have been synthesized using the constraint-synthesis method (Huang and Li 2002a, 2002b, 2002c). In addition, Frisoli et al. (2000) used the reciprocal screw theory to synthesize the 3-DoF translational PMs. Kong and Gosselin (2002) studied the type synthesis of 3-DoF spherical parallel manipulators.

Jin et al. (2001) proposed a motion-synthesis method based on the units of single-opened-chain and applied it to the type synthesis of 4-DoF PMs.

In this paper, first some important basic theoretical problems of the lower-mobility PMs are discussed. Then, we use the constraint-synthesis method to perform a systematic type synthesis of symmetrical 5-DoF and 4-DoF PMs. In addition, the characteristics of constraint and structure of 5-DoF and 4-DoF PMs are obtained in the process of type synthesis. Some novel 3-DoF PMs are proposed.

2. Basic Concepts and Some Problems

The constraint-synthesis method (Huang and Li 2002b) is based on screw theory (Ball 1900, Hunt 1978). The unit screw associated with a revolute pair or a force is given by $\$ = (\boldsymbol{s}; \boldsymbol{r} \times \boldsymbol{s}) = (l\ m\ n; a\ b\ c)$, where $\boldsymbol{s}$ is a unit vector along the screw axis, $\boldsymbol{r}$ is the position vector of any point on the screw axis, and l, m, n denote three direction cosines. The unit screw associated with a prismatic pair or a couple is given by $\$ = (0; \boldsymbol{s}) = (0\ 0\ 0; l\ m\ n)$.

2.1. Classification of Symmetrical Lower-mobility PMs

It is generally expected that a fully-symmetrical PM (Mohamed and Duffy 1984) can be obtained by type synthesis. A fully-symmetrical PM is one which satisfies all the following four conditions:

1. Having a full-cycle mobility;
2. Having identical limb kinematic chain;
3. Having all limbs arranged symmetrically on the base;
4. Having the same number and mounting position of actuators in every limb.

A mechanism-symmetrical PM is one which satisfies the first three conditions. An input-symmetrical PM is one which satisfies the first two and the fourth conditions. A limb-symmetrical PM is one which satisfies the first two conditions. An asymmetrical PM is one which only satisfies the first condition.

In addition, a PM with at least three rotational DoFs is called a fully-rotational PM, such as the 3-3R spherical PM and the 4-DoF PM proposed by Zlatanov and Gosselin (2001). A PM with at least three translational DoFs is called a fully-translational PM, such as the Delta robot, and the 4-DoF 4-URU PM proposed by Zhao and Huang (2000a).

2.2. Constraint-synthesis Method

In an earlier paper (Huang and Li 2002b), we proposed a general constraint-synthesis method. For readers' convenience, we give a brief review.

All the unit twists associated with the kinematic pairs in a limb form a limb twist system. A limb constraint system is a screw system formed by all wrenches reciprocal to the limb twists. The combination of all the limb constraints forms a mechanism constraint system.

The linearly independent unit twists representing the DoF of a PM form a mechanism twist system, which is reciprocal to the mechanism constraint system.

The key idea behind the constraint-synthesis method lies in the fact that for a lower-mobility PM, the combination of all the limb structural constraints determines what DoFs of the moving platform are constrained. Such a combined effect can be described by a mechanism constraint system. Based on the linear dependence of wrenches under different geometrical conditions, the desired limb constraint system can be obtained. Utilizing the reciprocal relation between wrench and twist, we can obtain a base of the desired limb twist system. Then, the desired limb kinematic chain can be generated by a linear combination of the twists in the base. Finally, following the geometrical condition which guarantees that the combination of all limb constraints is equal to the desired mechanism constraint system, we can use the desired limb kinematic chains to construct the desired PM.

Note that the linear combination of the twists in a base must maintain the linear independence between all twists in the base, which is called *linear independence rule*. The linear independence rule must be obeyed throughout this paper.

2.3. Identification of Instantaneous Mechanisms

In the type synthesis of the mechanism, one important problem is how to distinguish between instantaneous mechanisms and mechanisms of full-cycle mobility. Moreover, since twists and wrenches are instantaneous, it is also necessary to identify whether the synthesized mechanism is instantaneous.

This can be obtained by verifying the mechanism constraint system after any feasible finite displacement. If the mechanism constraint system remains unchanged, the mechanism is non-instantaneous. Such a verification can be done by simple analysis or inspection of geometrical conditions among the kinematic pairs in each limb and the mechanism after any given feasible finite displacement.

2.4. Input Selection Method

After the structural design of a new PM, it is necessary to choose a set of actuated pairs, the number of which is generally equal to the DoFs of the mechanism. However, this choice of actuated pairs cannot be arbitrary, or the mechanism cannot generate the desired motion, and more input interference may also occur.

The input selection method of lower-mobility PMs was first proposed by Zhao and Huang (2000b). The basic idea behind the input selection method is that when all the actuated pairs in a PM are locked, the moving platform loses all DoFs and cannot undergo any infinitesimal and finite motion. The rank of the mechanism constraint system is six. The procedure of input selection can be stated as follows.

Step 1 Write the limb twist system.

Step 2 Select a set of the actuated pairs and delete the twists corresponding to the actuated pairs in the limb twist system.

Step 3 Get the limb constraint system from the new reduced limb twist system.

Step 4 Combine all the limb constraint systems into the mechanism constraint system.

Step 5 Check the rank of the mechanism constraint system. If the rank is six, the input selection is right.

Note that for a lower-mobility parallel mechanism, generally there are several available sets of actuated pairs.

2.5. On the Motion-synthesis Method

The idea of the motion-synthesis method (Jin et al. 2001) is that the feasible motions/freedoms of the moving platform are the intersection of feasible motions/freedoms of all limbs, that is,

$$U^{\mathrm{P}} = \bigcap_{i=1}^{p} U_i = \begin{bmatrix} \bigcap_{i=1}^{p} \overline{x}_i & \bigcap_{i=1}^{p} \overline{y}_i & \bigcap_{i=1}^{p} \overline{z}_i \\ \bigcap_{i=1}^{p} \hat{x}_i & \bigcap_{i=1}^{p} \hat{y}_i & \bigcap_{i=1}^{p} \hat{z}_i \end{bmatrix}, \tag{1}$$

where U^{P} denotes the feasible motions of the moving platform, U_i is the feasible motions of the end of the ith limb, $\overline{x}_i$, $\overline{y}_i$, and $\overline{z}_i$ are three independent feasible translational DoFs, $\hat{x}_i$, $\hat{y}_i$, and $\hat{z}_i$ are three independent feasible rotational DoFs, and p is the number of limbs. It is worth indicating that the intersections of translational and rotational freedoms are performed separately and independently in eq (1).

It should be noted that, however, when some translational DoFs of the moving platform are constrained, the number of rotational freedoms may decrease. For example, two parallel constraint forces not only constrain a translational DoF along the forces, they also constrain a rotational freedom about the normal of the plane determined by the two forces; three coplanar and non-concurrent forces can also constrain a rotational freedom about the normal of the plane formed by these forces; three parallel and non-coplanar forces can constrain two rotational DoFs about the axes perpendicular to the forces (Huang, Tao and Fang 1996).

An example is given in what follows. The 3-DoF 3-RPS PM (Hunt 1983) has two rotational and one translational DoFs, as shown in Figure 1, where B denotes the base and M the moving platform. The end of a single RPS limb has five DoFs, including two translational DoFs and three rotational DoFs. The translational DoF along the first revolute axis is constrained. Obviously, the end of every RPS limb has three independent rotational DoFs. From eq (1), we have $\bigcap_{i=1}^{3} \hat{x}_i^l = \hat{x}$, $\bigcap_{i=1}^{3} \hat{y}_i^l = \hat{y}$, and $\bigcap_{i=1}^{3} \hat{z}_i^l = \hat{z}$. Finally, the intersection of U_1, U_2 and U_3 yields

$$U^{p} = \begin{bmatrix} 0 & 0 & \overline{z} \\ \hat{x} & \hat{y} & \hat{z} \end{bmatrix}. \tag{2}$$

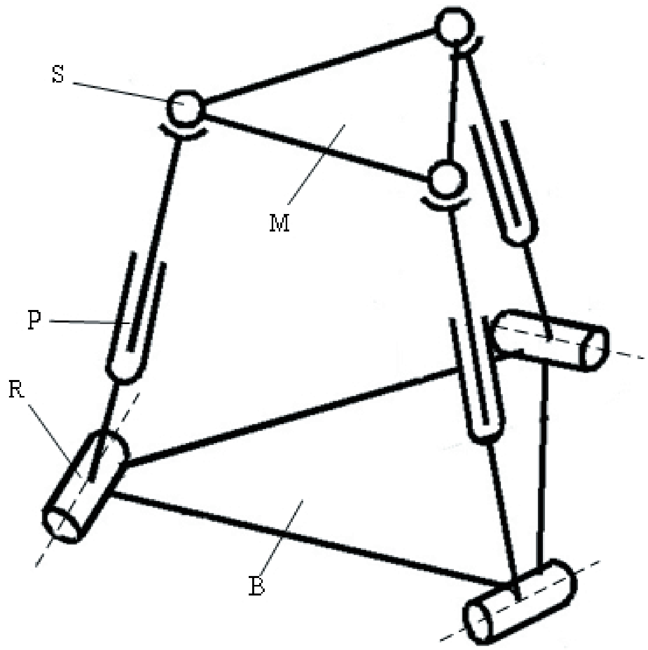

Fig. 1. 3-DoF 3-RPS PM.

Equation (2) shows that the feasible motion of the mechanism includes one translational DoF in the z-axis, and three rotational DoFs. This contradicts the well-established results of mobility of the 3-RPS PM (Hunt 1983, Huang, Tao and Fang 1996).

For the fully-translational mechanism, the limb constraint system and the mechanism constraint system only contain constraint couples, which are free vectors in space. The motion-synthesis method seems applicable to the fully-translational mechanism. Therefore the constraint-synthesis method is more general than the motion-synthesis method.

2.6. Nomenclature

In the following sections, some symbols are used to describe the limb kinematic chain. Generally, the left lowercase superscript is used to denote the direction of the axis of revolute or prismatic pair. The right capital subscript is used to denote the point on the pair axis if necessary. The left superscript z denotes that the axis of the pair is perpendicular to the base plane. The left superscript x or y denotes that the axis of the pair is parallel to the base plane. The left superscript u denotes that the axis of the pair bevels with the base plane in direction characterized by the unit vector u. We use u_i to denote different axes which also bevel with the base plane. The left superscripts i, j and k denote three linearly independent direction vectors. For example, $({}^{i}R{}^{j}R{}^{k}R)_N$ denotes three successive revolute pairs whose axes intersect at a common point N, namely, a 3R spherical subchain or a spherical pair. Another example is ${}^{ui}U_N$, where the superscripts u and i denote the first revolute pair axis and the second revolute pair axis of the universal joint; the subscript N indicates that the second revolute pair axis of the universal joint passes point N.

In addition, R also denotes rotational DoF and T denotes translational DoF. Lowercase right superscripts are used to denote the directions of the DoFs if necessary. For example, a $3R2T^{xy}$5-DoF PM refers to a PM with three rotational DoFs and two translational DoFs in the XY plane. It is also worth mentioning that for simplicity we neglect limb kinematic chains with helical pairs in this paper. If helical pairs are taken into consideration, more new architectures can be obtained.

For convenience, $\mathbf{\$}_{ij}$ is used to represent the unit twist associated with the jth kinematic pair in the ith limb; $\mathbf{\$}^r_{ij}$ the jth unit wrench exerted by the ith limb; $\mathbf{\$}_{mj}$ the jth unit twist in the mechanism twist system; $\mathbf{\$}^r_{mj}$ the jth unit wrench in the mechanism constraint system.

We set the XY plane of the global frame, $O-XYZ$, parallel to the base plane, thus the Z-axis is perpendicular to the base plane and upward. We set the z_i-axis of the ith limb frame, $o_i - x_i y_i z_i$, upward and perpendicular to the base plane. The limb twist and constraint system are expressed in the limb frame while the mechanism twist and constraint systems are expressed in the global frame.

2.7. Correct Application of the Grübler-Kutzbach Criterion

The general Grübler-Kutzbach criterion is of importance in the synthesis of lower-mobility PM. However, it has been difficult to correctly apply that criterion to lower-mobility PMs for a long time, especially to the PMs containing a closed loop in each limb. In the next two sections, this problem is discussed by constraint analysis.

Consider an M-DoF PM, comprising p limbs, each exerting q structural constraints on the moving platform. The $p \cdot q$ constraints form the mechanism constraint system, which must be a $6 - M$ system in the general configuration.

Because most of the lower-mobility PMs are over-constrained mechanisms, it is necessary to take the order of mechanism (Hunt 1978) and the redundant constraints into consideration. The order of a mechanism is given by

$$d = 6 - \lambda, \quad (3)$$

where λ is the number of the independent common constraints in the mechanism.

A common constraint is defined as a screw reciprocal to the unit twists associated with all kinematic pairs in a lower-mobility PM. A common constraint exists if and only if each limb provides one constraint of the same kind and all the p constraints are coaxial, namely, the p constraints form a 1-system.

A redundant constraint is one that is linearly dependent with other constraints. When l constraints do not form a common constraint and form a $k(k < l)$ system, $l - k$ redundant constraints exist.

Thus, the Grübler-Kutzbach criterion can be finally rewritten as (Huang, Kong and Fang 1997)

$$M = d(n - g - 1) + \mathbf{XXX}_{i=1}^{g} f_i + v, \tag{4}$$

where M denotes the mobility of the mechanism, d is the order of the mechanism, n is the number of links, g is the number of kinematic pairs, f_i is the freedom of the ith pair and v is the number of redundant constraints.

2.8. Generalized Pair and Mobility Analysis of PMs with a Closed-loop in Each Limb

It is shown that a planar four-bar parallelogram in a kinematic chain can be treated as a generalized prismatic pair (Huang and Li 2002b). For the PM with closed loops in the limbs, such as Delta robot (Clavel 1988) and Tsai's robot (Stamper, Tsai and Walsh 1997), the concept of a generalized kinematic pair will aid the mobility analysis.

Figure 2 shows a variant of the Delta robot (Clavel 1990), where a planar four-bar parallelogram with four universal joints in each vertex is in each limb. The 4U parallelogram may be treated as a generalized kinematic pair.

The 4U parallelogram can be considered as a PM with two 2U limbs, as shown in Figure 2(b). We take the output link 3 as the moving platform and the fixed link 1 as the base platform. Considering the structural condition of the 2U limb, we set the central point of the first universal joint as the origin of the limb frame, and the x-axis coincident with the first revolute axis and the y-axis coincident with the second revolute axis. Thus, the limb twist system of limb 1 in the general configuration is given by

$$\begin{aligned} \$_{11} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$_{12} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$_{13} &= (0\ 1\ 0\ ;\ -z_1\ 0\ x_1) \\ \$_{14} &= (1\ 0\ 0\ ;\ 0\ z_1\ 0), \end{aligned} \tag{5}$$

where $(x_1, 0, z_1)$ is the coordinate of the central point of the second universal joint in limb 1.

The limb constraint system of limb 1 is

$$\begin{aligned} \$^r_{11} &= (0\ 0\ 0\ ;\ 0\ 0\ 1) \\ \$^r_{12} &= (x_1\ 0\ z_1\ ;\ 0\ 0\ 0), \end{aligned} \tag{6}$$

which is one constraint couple perpendicular to the first universal joint plane and a constraint force along the link 2.

Thus, limbs 1 and 2 exert four constraints in total on the output link 3. The two couples are coaxial and restrict one rotational DoF about the z-axis, namely, the normal of the first universal joint plane. The two forces are parallel and restrict a translational DoF along link 2 and a rotational DoF about the normal of the plane formed by links 2 and 4.

Thus, the 4U parallelogram can be regarded as a generalized kinematic pair, denoted by a superscript g, with three DoFs:

$$\begin{aligned} \$^g_1 &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$^g_2 &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\ \$^g_3 &= (0\ 0\ 0\ ;\ -z_1\ 0\ x_1), \end{aligned} \tag{7}$$

which express one rotational and two translational DoFs.

When analyzing the mobility of such a robot, we replace the 4U parallelogram by the generalized kinematic pair. Thus, such a limb consists of three links and four kinematic pairs, as shown in Figure 2(c).

The limb twist system of the robot is a 4-system, that is,

$$\begin{aligned} \$^g_{i1} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$^g_{i2} &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\ \$^g_{i3} &= (0\ 0\ 0\ ;\ -z_1\ 0\ x_1) \\ \$^g_{i4} &= (0\ 1\ 0\ ;\ a_4\ 0\ c_4). \end{aligned} \tag{8}$$

The limb constraint system is

$$\begin{aligned} \$^r_{i1} &= (0\ 0\ 0\ ;\ 1\ 0\ 0) \\ \$^r_{i2} &= (0\ 0\ 0\ ;\ 0\ 0\ 1), \end{aligned} \tag{9}$$

which shows that each limb exerts two constraint couples on the moving platform, one being perpendicular to the base plane and the other being parallel to the base plane.

The three limbs of the robot exert six constraint couples in total on the moving platform. The three constraint couples perpendicular to the base plane form a common constraint and $\lambda = 1$. The other three constraint couples are coplanar and only two of them are linearly independent, thus forming a 2-system. Hence, one redundant constraint exists and $\nu = 1$. Accordingly, there are three linearly independent constraint couples, thereby constraining three rotational DoFs. The mechanism has three translational freedoms. Using eq (4), we have

$$M = 5(11 - 12 - 1) + 12 + 1 = 3. \tag{10}$$

Similarly, for Tsai's mechanism as shown in Figure 3, the four-bar parallelogram in each limb with four revolute joints in each vertex corresponds to a 1-DoF generalized translational pair.

Thus, the limb twist system in the general non-singular configuration is given by

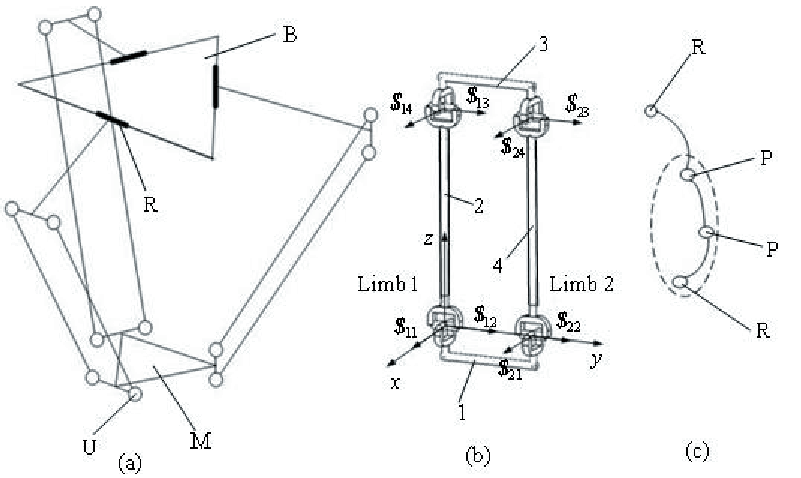

Fig. 2. A variant of the Delta parallel robot.

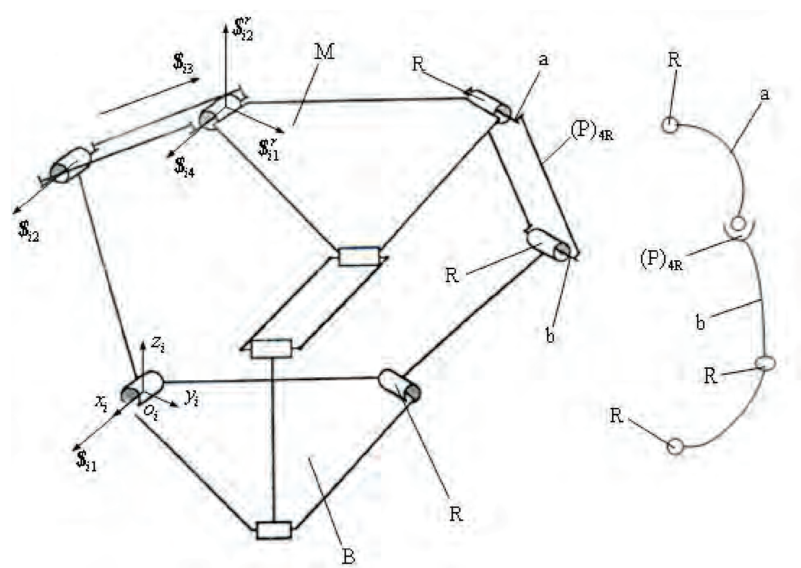

Fig. 3. Tsai’s mechanism.

$$\begin{aligned}
\$_{i1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\
\$_{i2} &= (1\ 0\ 0\ ;\ 0\ b_2\ c_2) \\
\$_{i3}^{g} &= (0\ 0\ 0\ ;\ l_3\ m_3\ n_3) \\
\$_{i4} &= (1\ 0\ 0\ ;\ 0\ b_4\ c_4).
\end{aligned} \tag{11}$$

The limb constraint system is

$$\begin{aligned}
\$_{l1}^{r} &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\
\$_{l2}^{r} &= (0\ 0\ 0\ ;\ 0\ 0\ 1).
\end{aligned} \tag{12}$$

Equation (12) shows that a single limb in Tsai's mechanism produces the same constraints as those in the variant of Delta robot on the moving platform. Thus, the Tsai's mechanism is a 3-DoF translational mechanism, and the mobility is also given by eq (10).

The above two sections show that it is effective to use the constraint analysis to aid the mobility analysis by the general Grübler-Kutzbach criterion. Not only the number of the mobility of lower-mobility PMs can be obtained, but the properties of DoFs, namely, rotational or translational, are also available.

3. Type Synthesis of 5-DoF PMs

The 5-DoF PMs fall into two categories according to their mobility: one category has three rotational DoFs and two translational DoFs; the other has three translational DoFs and two rotational DoFs.

The five DoFs of the mechanism can be represented by five linearly independent twists, which form a 5-system. Clearly, there exists only one screw reciprocal to the 5-system, namely, $q = 1$, which forms the mechanism constraint system and means that the moving platform loses only one freedom. Thus, the maximum linearly independent number of the p limb constraints must be one, i.e., they form a 1-system. In terms of geometry, the p structure constraints must be coaxial, forming a common constraint.

Consequently, in such a 5-DoF PM, we can find $\lambda = 1$ and $\nu = 0$. Using eq (4), we have

$$M = 5(n - g - 1) + \sum_{i=1}^{g} f_i. \tag{13}$$

3.1. 5-DoF PMs with Three Rotational DoFs and Two Translational DoFs in the XY Plane

3.1.1. Procedure of Type Synthesis

For simplicity, we focus on the 5-DoF PM with three rotational DoFs and two translational DoFs in the XY plane, denoted by 3R2T^{xy}. The standard base of the mechanism twist system is given by

$$\begin{aligned}
\$_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\
\$_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\
\$_{m3} &= (0\ 0\ 1\ ;\ 0\ 0\ 0) \\
\$_{m4} &= (0\ 0\ 0\ ;\ 1\ 0\ 0) \\
\$_{m5} &= (0\ 0\ 0\ ;\ 0\ 1\ 0).
\end{aligned} \tag{14}$$

The standard base of the mechanism constraint system is given by

$$\$_{m1}^{r} = (0\ 0\ 1\ ;\ 0\ 0\ 0). \tag{15}$$

Using eq (14) as the standard base of the limb twist system, we can obtain the limb kinematic chain by linear combination of the five twists. It is worth recalling that the linear independence rule must be obeyed. In this case, the standard base of the limb constraint system is also given by eq (15).

The linear combination of $\$_{m4}$ and $\$_{m5}$ only yields prismatic pairs parallel to the XY plane. By linear combination with $\$_{m3}$, they also can be transformed into revolute pairs whose axes are in the z_i-axis direction. Thus, the limb kinematic chain can consist of five revolute pairs.

As for $\$_{m1}$, $\$_{m2}$ and $\$_{m3}$, they themselves can represent a spherical pair or three successive revolute pairs whose axes intersect at a common point, namely, a 3R spherical subchain. The common point is called the limb central point. If by linear combination with $\$_{m4}$ and $\$_{m5}$, $\$_{m3}$ can be transformed into $\$'_{m3} = (0\ 0\ 1\ ;\ a_3\ b_3\ 0)$, which denotes a revolute pair whose axis is in the z_i-axis direction and does not pass the limb central point. Consequently, the remaining $\$_{m1}$ and $\$_{m2}$ form a 2R spherical subchain.

In order to keep the mechanism constraint system unchangeable when the mechanism moves, we should keep the sixth component in any twist in eq (14) to be zero and also unchangeable. Therefore, all the axes of revolute pairs are divided into two groups. The axes in one group are perpendicular to the base. The axes in the other group successively intersect at a common point, which is selected as the global origin. The number of the axes either normal to the base or intersecting at a common point cannot be greater than three. Otherwise some pairs in the limb will be linearly dependent. Thus, the mechanism must contain a 3R or 2R spherical subchain. The axes of revolute pairs outside the 2R or 3R spherical subchain must be perpendicular to the base plane. For the same reason, the orientation of any translational pair should be parallel to the base.

Because force is not a free vector in space, a point on the force axis is necessary to determine a force vector in space. Several forces are coaxial if and only if they pass a common point and are parallel to each other. Because each limb has a limb central point passed through by the constraint force, all the limb central points must coincide so that the limb constraint forces are coaxial. Such a superposition point is called a mechanism central point. The mechanism central point is fixed relative to the base or the moving platform because of the symmetrical structure.

Since the center of all the spherical pairs in all limbs cannot coincide with each other in general, the limb kinematic chain contains no spherical pairs. In addition, all the 2R or 3R subchains must connect either to the base or to the moving platform simultaneously to avoid being instantaneous.

Based on the above analysis, the characteristics of constraint and structure of such 5-DoF PMs are shown in Table A1 in the Appendix. Furthermore, we can obtain the usable limb kinematic chains by a linear combination of the five twists in eq (14).

An enumeration of the PMs is given in Table 1, in which $2 \leq p \leq 5$. Note that cylindrical pairs and universal joints are kinematically equal to a specific combination of revolute pairs and prismatic pairs. By setting the first revolute pair axis of the 3R or 2R spherical subchain perpendicular to its anterior revolute pair axis and assuming the intersection of the two axes, we can obtain a universal joint. Similarly, by setting the first revolute pair axis of the 3R or 2R spherical subchain parallel to its anterior prismatic pair, we can obtain a cylindrical pair.

3.1.2. Examples

A $3-{}^{x}P^{z}R^{z}R({}^{i}R^{j}R)_{N}$ PM is shown in Figure 4. Obviously, the mechanism is mechanism-symmetrical. Following the steps in Section 2.4, we can obtain the appropriate input selection. Any two prismatic pairs and one revolute pairs in each 2R subchain can be chosen as an actuated pair. Figure 5 shows another 5-DoF $3-{}^{x}P^{z}R({}^{i}R^{j}R^{k}R)_{N}$ PM. The two mechanisms are not instantaneous.

Applying eq (13) to the above two mechanism, we have

$$M = 5(14 - 15 - 1) + 15 = 5. \tag{16}$$

Table 1. Symmetrical 3R2T^{xy} 5-DoF PMs

With 2R Spherical Subchain	With 3R Spherical Subchain
$p-{}^{z}R^{z}R^{z}R({}^{i}R^{j}R)_{N}$	
$p-{}^{z}R^{z}R^{x}P({}^{i}R^{j}R)_{N}$	$p-{}^{z}R^{z}R({}^{i}R^{j}R^{k}R)_{N}$
$p-{}^{z}R^{x}P^{z}R({}^{i}R^{j}R)_{N}$	$p-{}^{x}P^{y}P({}^{i}R^{j}R^{k}R)_{N}$
$p-{}^{x}P^{z}R^{z}R({}^{i}R^{j}R)_{N}$	$p-{}^{z}R^{x}P({}^{i}R^{j}R^{k}R)_{N}$
$p-{}^{x}P^{y}P^{z}R({}^{i}R^{j}R)_{N}$	$p-{}^{x}P^{z}R({}^{i}R^{j}R^{k}R)_{N}$
$p-{}^{z}R^{x}P^{y}P({}^{i}R^{j}R)_{N}$	
$p-{}^{x}P^{z}R^{y}P({}^{i}R^{j}R)_{N}$	

3.2. 5-DoF PMs with Two Rotational DoFs and Three Translational DoFs

3.2.1. Procedure of Type Synthesis

The 5-DoF PM with three translational DoFs and two rotational DoFs is denoted by 2R3T. We assume that the two rotational axes are in the XY plane in the initial configuration. The standard base of the mechanism twist system is given by

$$\begin{aligned} \mathbf{\$}_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \mathbf{\$}_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \mathbf{\$}_{m3} &= (0\ 0\ 0\ ;\ 1\ 0\ 0) \\ \mathbf{\$}_{m4} &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\ \mathbf{\$}_{m5} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \tag{17}$$

The standard base of the mechanism constraint system is given by

$$\mathbf{\$}^{r}_{m1} = (0\ 0\ 0\ ;\ 0\ 0\ 1). \tag{18}$$

Using eq (17) as the standard base of the limb twist system, we can obtain the limb kinematic chain by a linear combination of the five twists. Note that the standard base of the limb constraint system is also given by eq (18) in this case.

The linear combination of $\mathbf{\$}_{m3}$, $\mathbf{\$}_{m4}$, and $\mathbf{\$}_{m5}$ yields a prismatic pair which bevels with the base. Obviously, $\mathbf{\$}_{m3}$, $\mathbf{\$}_{m4}$ and $\mathbf{\$}_{m5}$ can be transformed into revolute pairs whose axes are parallel to the XY plane by linear combination with $\mathbf{\$}_{m1}$ or $\mathbf{\$}_{m2}$. Thus, all revolute axes must be parallel to the XY plane in the initial configuration. This also means that the limb kinematic chain contains no spherical pairs.

It should be noted, however, that the limb constraint couple is perpendicular to the plane formed by $\mathbf{\$}_{m1}$ and $\mathbf{\$}_{m2}$. Every limb has such a plane determining the direction of the constraint couple. These planes must always be parallel to each other to guarantee that the constraint couples are in the same direction. To meet this condition, the axes of revolute pairs most adjacent to the base in all limbs must be set parallel and so are the axes of the revolute pairs most adjacent to the moving platform. Because of this, the mechanism of this type is only limb-symmetrical.

The characteristics of constraint and structure of such 5-DoF PMs are shown in Table A2 in the Appendix. Accordingly, we can obtain the limb kinematic chain by linear combination of the five twists in eq (17). The enumeration of such PMs is shown in Table 2.

3.2.2. Examples

Consider a ${}^{x}R^{u}P^{x}R^{y}R^{y}R$ limb. Note that ${}^{x}R^{y}R$ can form a universal joint ${}^{xy}U_{N}$. Figure 6 shows such a 5-DoF $5-{}^{x}R^{u}P^{xy}U_{N}{}^{y}R$ PM. The universal joint plane in the ith limb is denoted by U_{34},

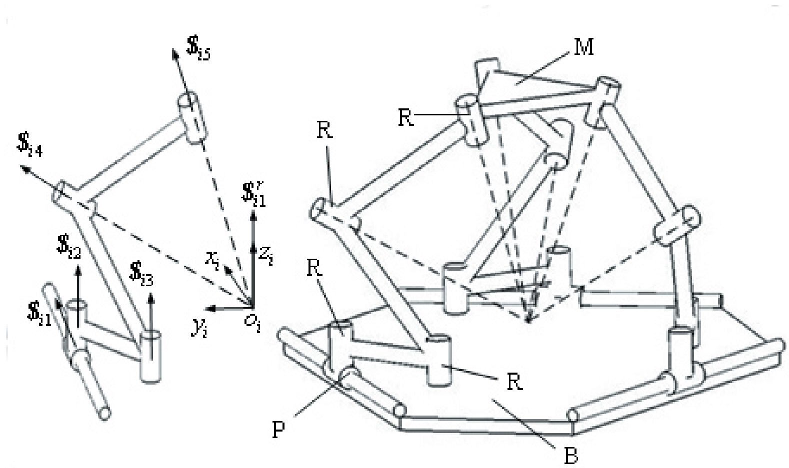

Fig. 4. $3R2T^{xy}$ 5-DoF $3 - {}^{x}P^{z}R^{z}R({}^{i}R^{j}R)_{N}$ PM.

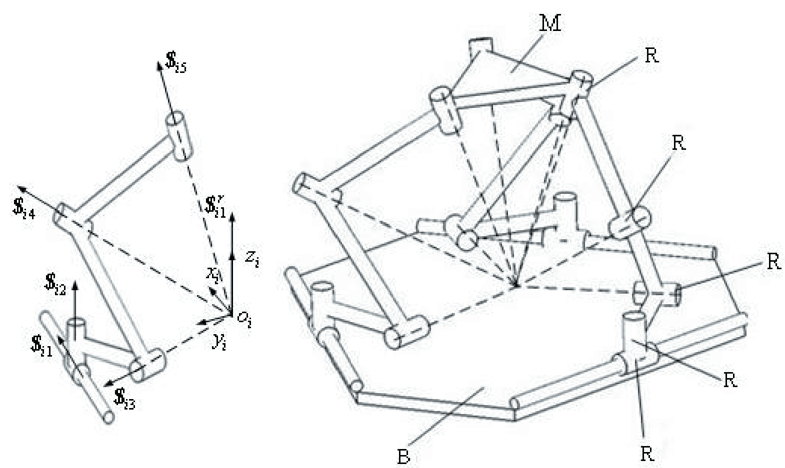

Fig. 5. $3R2T^{xy}$ 5-DoF $n - {}^{x}P^{z}R({}^{i}R^{j}R^{k}R)_{N}$ PM.

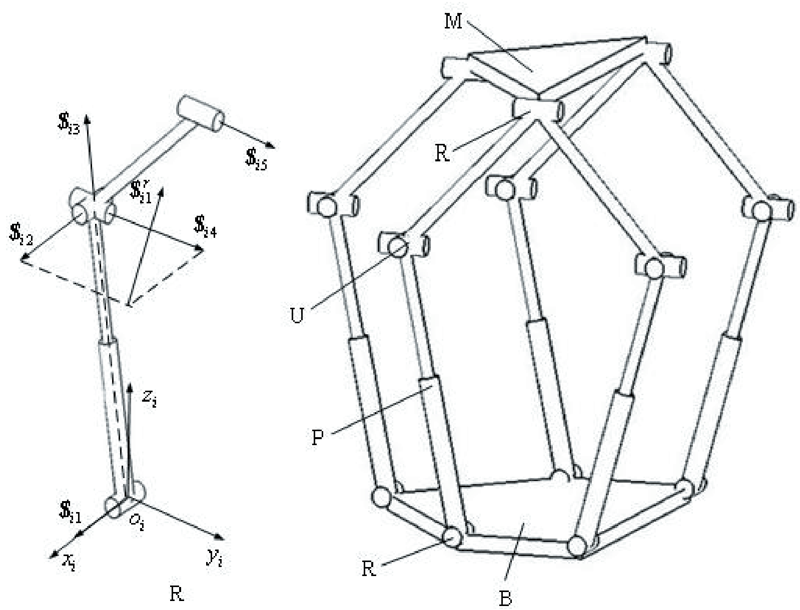

Fig. 6. 2R3T 5-DoF $5-{}^{x}R^{u}P^{xy}U_{N}{}^{y}R$ PM.

Table 2. Symmetrical 2R3T 5-DoF PMs

With No or One Prismatic Pair	With Two Prismatic Pairs
$p-{}^{x}R^{x}R^{x}R^{y}R^{y}R$	$p-{}^{u}P^{u}P^{x}R^{x}R^{y}R$
$p-{}^{u}P^{x}R^{x}R^{y}R^{y}R$	$p-{}^{u}P^{u}P^{x}R^{y}R^{y}R$
$p-{}^{x}R^{u}P^{x}R^{y}R^{y}R$	$p-{}^{x}R^{u}P^{u}P^{y}R^{y}R$
$p-{}^{x}R^{x}R^{u}P^{y}R^{y}R$	$p-{}^{u}P^{x}R^{u}P^{y}R^{y}R$
$p-{}^{x}R^{x}R^{y}R^{u}P^{y}R$	$p-{}^{x}R^{u}P^{y}R^{u}P^{y}R$

which is parallel to the base plane in the initial configuration. The geometrical arrangement of the five limbs guarantees that the five universal joint planes are always parallel.

In this initial configuration, such a single ${}^{x}R^{u}P^{xy}U_{N}{}^{y}R$ limb exerts a constraint couple as that in eq (18) on the moving platform and restricts the rotation about the normal of U_{34}. Because the five constraint couples are parallel, they are linearly dependent and form a 1-system. The mechanism constraint system is still the same as that in eq (18).

Note that after the moving platform undergoes arbitrary translation or rotation about the y_i-axis, the limb twist system remains unchanged and the plane of U_{34} is always parallel to the base plane. The mechanism constraint system remains the same as that in eq (18).

After the moving platform undergoes arbitrary finite rotation about the x_i-axis, the limb twist system becomes

$$\begin{aligned}
\$_{i1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0)\\
\$_{i2} &= (0\ 0\ 0\ ;\ 0\ m_2\ n_2)\\
\$_{i3} &= (1\ 0\ 0\ ;\ 0\ b_3\ c_3)\\
\$_{i4} &= (0\ m_4\ n_4\ ;\ a_4\ b_4\ c_4)\\
\$_{i5} &= (0\ m_4\ n_4\ ;\ a_5\ b_5\ c_5).
\end{aligned} \tag{19}$$

The limb constraint system becomes

$$\$^r_{i1} = (0\ 0\ 0\ ;\ 0\ -n_4\ m_4). \tag{20}$$

$\$^r_{i1}$ denotes a constraint couple perpendicular to U_{34}. Since all five universal joint planes of U_{34} are parallel, the five limb constraint couples are parallel and linearly dependent, thereby equaling one couple, namely, $\$^r_{i1}$.

In brief, the mechanism constraint system of the $5-{}^{x}R^{u}P^{xy}U_{N}{}^{y}R$ PM only contains one constraint couple along the normal of the plane of U_{34}. Thus, the mechanism loses a rotational DoF about the normal of U_{34} and has three translational DoFs and two rotational DoFs. From the above analysis, it can be seen that the mechanism constraint system remains unchanged after any finite non-singular displacement. Hence, the mechanism is not instantaneous.

Using eq (13), we have

$$M = 5(22-25-1)+25 = 5. \tag{21}$$

Using the input selection method in Section 2.4, we can find that the five prismatic pairs can be chosen as inputs.

4. Type Synthesis of 4-DoF PMs

The 4-DoF PMs fall into three categories according to their mobility. The first category has three rotational DoFs and one translational DoF. The second has three translational DoFs and one rotational DoF. The third has two rotational DoFs and two translational DoFs.

The four DoFs of the mechanism can be represented by four linearly independent twists, which form a 4-system. Clearly, there exist only two screws reciprocal to the 4-system, which forms the mechanism constraint system and means the moving platform loses only two freedoms. Thus, the maximum linearly independent number of the $p \cdot q$ limb constraints must be two, i.e., they form a 2-system.

4.1. 4-DoF Parallel Mechanisms with Three Translational DoFs and One Rotational DoF

4.1.1. Procedure of Type Synthesis

For simplicity, we focus on the 4-DoF PM with three translational DoFs and one rotational DoF about the Z-axis, denoted by 3TIR^z. The standard base of the mechanism twist system is given by

$$
\begin{aligned}
\$_{m1} &= (0\ 0\ 1\ ;\ 0\ 0\ 0)\\
\$_{m2} &= (0\ 0\ 0\ ;\ 1\ 0\ 0)\\
\$_{m3} &= (0\ 0\ 0\ ;\ 0\ 1\ 0)\\
\$_{m4} &= (0\ 0\ 0\ ;\ 0\ 0\ 1).
\end{aligned}
\tag{22}
$$

The standard base of the mechanism constraint system is given by

$$
\begin{aligned}
\$^r_{m1} &= (0\ 0\ 0\ ;\ 1\ 0\ 0)\\
\$^r_{m2} &= (0\ 0\ 0\ ;\ 0\ 1\ 0).
\end{aligned}
\tag{23}
$$

Case 1. The limb kinematic chain consists of four kinematic pairs and exerts two constraint couples on the moving platform.

In this case, the standard base of the limb twist system is the same as eq (22). We can obtain the limb kinematic chain by linear combination of the four twists. Only two prismatic pairs $\$_{m2}$ and $\$_{m3}$ can be transformed into revolute pairs by linear combination with $\$_{m1}$, thus producing two revolute pairs in the z_i-axis direction.

Case 2. The limb kinematic chain consists of five kinematic pairs and exerts one constraint couple on the moving platform.

In this case, the limb kinematic chain only exerts one constraint couple on the moving platform. We need to add one twist $\$_{i5} = (1\ 0\ 0\ ;\ 0\ 0\ 0)$ to eq (22) to eliminate one corresponding constraint. The new limb twist system is

$$
\begin{aligned}
\$_{i1} &= (0\ 0\ 1\ ;\ 0\ 0\ 0)\\
\$_{i2} &= (0\ 0\ 0\ ;\ 1\ 0\ 0)\\
\$_{i3} &= (0\ 0\ 0\ ;\ 0\ 1\ 0)\\
\$_{i4} &= (0\ 0\ 0\ ;\ 0\ 0\ 1)\\
\$_{i5} &= (1\ 0\ 0\ ;\ 0\ 0\ 0).
\end{aligned}
\tag{24}
$$

The limb constraint system reciprocal to eq (24) is

$$
\$^r_{i1} = (0\ 0\ 0\ ;\ 0\ 1\ 0), \tag{25}
$$

which is a constraint couple in the y_i-axis. Thus, all the limb constraint couples are parallel to the XY plane.

$\$_{i2}$, $\$_{i3}$ and $\$_{i4}$ can be transformed into revolute pairs by linear combination with $\$_{i1}$ or $\$_{i5}$. Thus, the revolute pairs fall into two groups. The axes of one group are parallel to the base plane. The axes of the other group are perpendicular to the base plane. To prevent the mechanism from being instantaneous, the axes of revolute pairs fixed to the base or the moving platform must be perpendicular to the base, as shown in Figure 7.

Note that $\$^r_{i1}$ is a couple which is actually perpendicular to plane formed by $\$_{i1}$ and $\$_{i5}$, namely, the $x_i z_i$ plane. Since all the z_i-axes are parallel, we only need to make the x_i-axes of local systems not parallel to one another. Hence, the revolute axes parallel to the base plane in each limb must not be parallel to one another.

The characteristics of constraint and structure of such 3TIR^z 4-DoF PMs are shown in Table A3 in the Appendix. An enumeration of such 4-DoF PMs is shown in Table 3, where g/p denotes the number of kinematic pairs in a limb and $2 \le p \le 4$.

4.1.2. Example

Consider a ^{z}R^{x}R^{u}P^{x}R^{z}R limb. ^{z}R^{x}R can form a universal joint ^{zx}U$_N$ while ^{x}R^{z}R can form a universal joint ^{xz}U$_M$. Figure 7 shows a $4-{}^{zx}$U$_N{}^u$P^{xz}U$_M$ PM. Each ^{zx}U$_N{}^u$P^{xz}U$_M$ branch exerts a constraint couple on the moving platform. This couple is perpendicular to the first universal joint plane. All four limbs exert four constraint couples on the moving platform in total. Because the four constraint couples are coplanar, only two of them are linearly independent and two redundant constraints exist, i.e., $\nu = 2$. The two independent constraint couples restrict two rotational DoF of the moving platform about the axes parallel to the base plane. The remaining four DoFs of the moving platform include three translational DoFs and one rotational DoF about the normal of the base plane.

Because the constraint couples are not coaxial, $\lambda = 0$. Using eq (4), we have

$$
M = 6(18 - 20 - 1) + 20 + 2 = 4. \tag{26}
$$

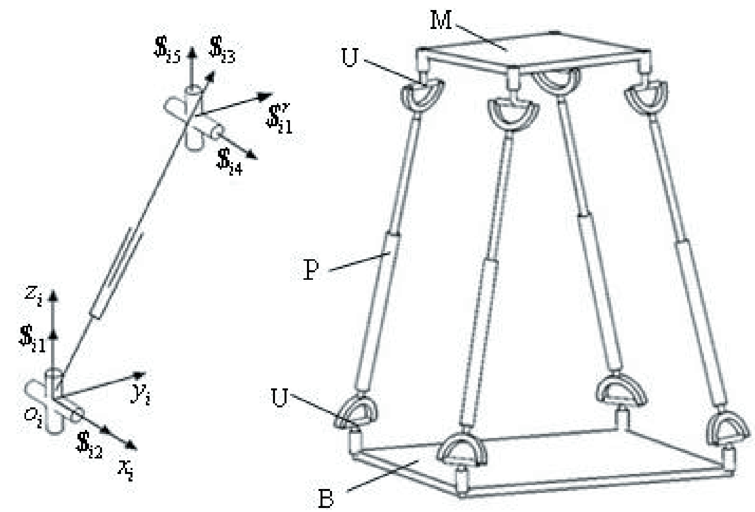

Fig. 7. 3TIR^z 4-DoF $4-{}^{zx}U_N{}^{u}P^{xz}U_M$ PM.

Table 3. Symmetrical 3TIR^z 4-DoF PMs

g/p	PMs		
4	$p-{}^{z}R^{i}P^{j}P^{k}P$	$p-{}^{i}P^{z}R^{j}P^{k}P$	$p-{}^{z}R^{j}P^{z}R^{z}R$
	$p-{}^{z}R^{z}R^{i}P^{j}P$	$p-{}^{z}R^{i}P^{z}R^{j}P$	$p-{}^{i}P^{j}P^{z}R^{k}P$
	$p-{}^{i}P^{z}R^{z}R^{j}P$	$p-{}^{z}R^{z}R^{j}P^{z}R$	$p-{}^{z}R^{i}P^{j}P^{z}R$
	$p-{}^{j}P^{z}R^{z}R^{z}R$		
5	$p-{}^{z}R^{z}R^{z}R^{x}R^{x}R$	$p-{}^{x}R^{x}R^{z}R^{z}R^{u}P$	
	$p-{}^{x}R^{x}R^{x}R^{z}R^{z}R$	$p-{}^{u}P^{z}R^{x}R^{x}R^{x}R$	
	$p-{}^{z}R^{x}R^{x}R^{x}R^{z}R$	$p-{}^{z}R^{u}P^{x}R^{x}R^{x}R$	
	$p-{}^{u}P^{z}R^{z}R^{x}R^{x}R$	$p-{}^{u}P^{x}R^{z}R^{z}R^{z}R$	
	$p-{}^{z}R^{z}R^{u}P^{x}R^{x}R$	$p-{}^{x}R^{u}P^{z}R^{z}R^{z}R$	
	$p-{}^{z}R^{z}R^{x}R^{x}R^{u}P$	$p-{}^{u}P^{x}R^{x}R^{x}R^{z}R$	
	$p-{}^{u}P^{x}R^{x}R^{z}R^{z}R$	$p-{}^{x}R^{x}R^{x}R^{u}P^{z}R$	
	$p-{}^{x}R^{x}R^{u}P^{z}R^{z}R$	$p-{}^{u}P^{z}R^{z}R^{z}R^{x}R$	
	$p-{}^{z}R^{x}R^{u}P^{x}R^{z}R$	$p-{}^{z}R^{z}R^{z}R^{u}P^{x}R$	
		$p-{}^{z}R^{u}P^{z}R^{x}R^{x}R$	
		$p-{}^{x}R^{u}P^{x}R^{z}R^{z}R$	

Note that, after any feasible finite motion, the two universal joint planes in each branch remain perpendicular to the base plane since the moving platform can only translate in space and rotate about the normal of itself. Thus, the mechanism constraint system remains unchanged and the mechanism is non-instantaneous.

4.2. 4-DoF PMs with Three Rotational DoFs and One Translational DoF

4.2.1. Procedure of Type Synthesis

For simplicity, we focus on the 4-DoF PM with three rotational DoFs and one translational DoF along the Z-axis, which is denoted by 3RIT^z. The standard base of the mechanism twist system is given by

$$\begin{aligned} \$_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$_{m3} &= (0\ 0\ 1\ ;\ 0\ 0\ 0) \\ \$_{m4} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \tag{27}$$

The standard base of the mechanism constraint system is given by

$$\begin{aligned} \$^r_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$^r_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0). \end{aligned} \tag{28}$$

Case 1. The limb kinematic chain consists of four kinematic pairs and exerts two constraint forces on the moving platform.

In this case, the standard base of the limb twist system is the same as eq (27). The linear combination of $\$_{m1}$, $\$_{m2}$ and $\$_{m3}$ can only produce a 3R spherical subchain. A 2R spherical subchain, however, will lead to an instantaneous mechanism. Note that the three limb central points must coincide with one another.

Case 2. The limb kinematic chain consists of five kinematic pairs and exerts one constraint force on the moving platform.

In this case, adding a twist $\$_{i5} = (0\ 0\ 0\ ;\ 0\ 1\ 0)$ to eq (27) yields a new limb twist system

$$\begin{aligned} \$_{i1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$_{i2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$_{i3} &= (0\ 0\ 1\ ;\ 0\ 0\ 0) \\ \$_{i4} &= (0\ 0\ 0\ ;\ 0\ 0\ 1) \\ \$_{i5} &= (0\ 0\ 0\ ;\ 0\ 1\ 0). \end{aligned} \tag{29}$$

The limb constraint system reciprocal to eq (29) is

$$\$^r_{i1} = (1\ 0\ 0\ ;\ 0\ 0\ 0), \tag{30}$$

which is a constraint force in the x_i-direction, passing through the limb central point.

Similar to the $3R2T^{xy}$ PMs, the limb kinematic chain must include a 2R or 3R spherical subchain. The revolute axes except those in the 2R or 3R subchain must be parallel to the base plane. Additionally, the linear combination of $\$_{i4}$ and $\$_{i5}$ with themselves only leads to a prismatic pair in the yz plane. Thus, if the limb contains prismatic pairs, they must be perpendicular to $\$_{i1}$.

The limb constraint is a force parallel to the revolute axes outside the spherical subchain, passing through the limb central point. When all the limb constraint forces are coplanar and passing through a common point, they form a desired 2-system as shown in eq (28).

Based on the above analysis, we can obtain the characteristics of constraint and structure of such 4-DoF PMs, as shown in Table A4 in the Appendix. The enumeration of such 4-DoF PMs is shown in Table 4, in which $2 \le p \le 4$.

Actually, the $4 - {}^{x}R{}^{x}R({}^{i}R{}^{j}R{}^{k}R)_N$ PM was first proposed by Zaltnov and Gosselin (2001).

4.2.2. Example

Consider a ${}^{x}R{}^{u}P{}^{x}R({}^{i}R{}^{j}R)_N$ limb. ${}^{x}R{}^{i}R$ can form a universal joint ${}^{xi}U_N$. Figure 8 shows a $4 - {}^{x}R{}^{u}P{}^{xi}U_N{}^{j}R_N$ PM. The constraint force $\$^r_{i1}$ passes through the mechanism central point and is parallel to the first revolute axis $\$_{i1}$. The four limbs exert four constraint forces on the moving platform. These forces are coplanar and always parallel to the base plane, intersecting at the mechanism central point. Thus, the standard base of the mechanism constraint system is the same as eq (28) and restricts the two translational DoFs parallel to the base plane. Similarly, we can identify that this mechanism is not instantaneous and has finite mobility.

Because these constraints are not coaxial, we have $\lambda = 0$. Only two of the four constraint forces are linearly independent, and we have $\nu = 2$.

Using eq (4), we have

$$M = 6(18 - 20 - 1) + 20 + 2 = 4. \tag{31}$$

The four prismatic pairs can be chosen as actuated pairs and this mechanism is fully-symmetrical.

Table 4. Symmetrical $3RIT^z$ 4-DoF PMs

g/p	PMs	
4	$p - {}^{z}P({}^{i}R{}^{j}R{}^{k}R)_N$	
5	$p - {}^{x}R{}^{x}R{}^{x}R({}^{i}R{}^{j}R)_N$	$p - {}^{x}R{}^{x}R({}^{i}R{}^{j}R{}^{k}R)_N$
	$p - {}^{u}P{}^{x}R{}^{x}R({}^{i}R{}^{j}R)_N$	$p - {}^{x}R{}^{u}P{}^{x}R({}^{i}R{}^{j}R)_N$
	$p - {}^{x}R{}^{x}R{}^{u}P({}^{i}R{}^{j}R)_N$	$p - {}^{u}P{}^{x}R({}^{i}R{}^{j}R{}^{k}R)_N$
	$p - {}^{x}R{}^{u}P({}^{i}R{}^{j}R{}^{k}R)_N$	

4.3. 4-DoF PMs with Two Rotational DoFs and Two Translational DoFs

In this case, the mechanism constraint system contains one force and one couple, restricting one translational freedom and one rotational freedom. However, we have not yet found such a non-instantaneous symmetrical parallel mechanism with full-cycle mobility .

5. Type Synthesis of 3-DoF PMs

The 3-DoF PM has been studied extensively and many novel 3-DoF PMs have been invented (Hervé and Sparacino 1991, Tsai 1999, Karouia and Hervé 2000, Di Gregorio 2001, 2002, Carricato and Parenti-Castelli 2002). Here we only give a brief investigation using our method for systematization and completeness. It is worth mentioning that some new 3-DoF PMs can still be found using our method.

The 3-DoF PMs fall into four categories according to their mobility. The first category has three rotational DoFs. The second has three translational DoFs. The third has two rotational DoFs and one translational DoF, such as the 3-RPS PM (Hunt 1983). The fourth has two translational DoFs and one rotational DoF, which is kinematically equal to a planar 8-bar PM.

The three DoFs can be represented by three linearly independent twists, which form a 3-system. Clearly, there exist only three screws reciprocal to the 3-system, which form the mechanism constraint system. Thus, the maximum linearly independent number of the $p \cdot q$ limb constraints must be three, i.e., they form a 3-system.

5.1. 3-DoF PMs with Three Rotational DoFs

Consider the 3-DoF rotational PM. The standard base of the mechanism twist system is

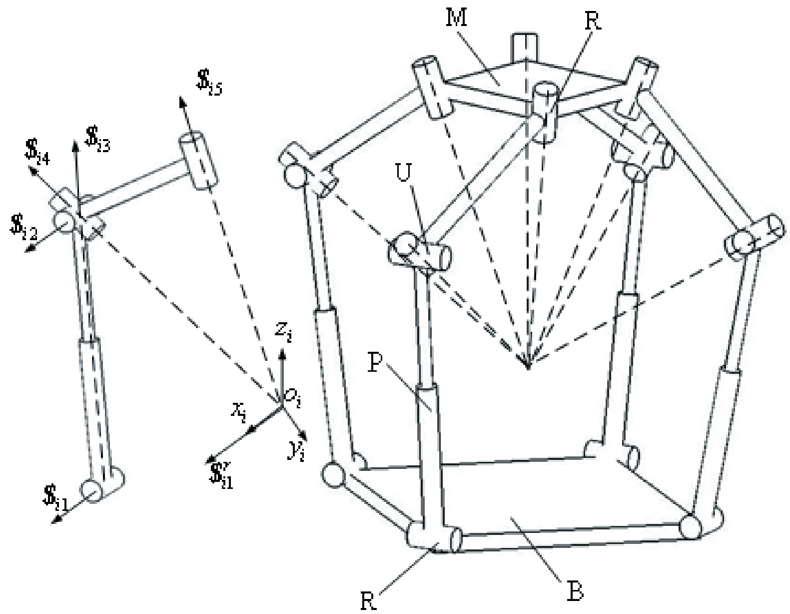

Fig. 8. 3RIT^z 4-DoF 4 – $^xR^uP^{xi}U_N{}^jR_N$ mechanism.

$$\$_{m1} = (1\ 0\ 0\ ;\ 0\ 0\ 0)$$
$$\$_{m2} = (0\ 1\ 0\ ;\ 0\ 0\ 0) \tag{32}$$
$$\$_{m3} = (0\ 0\ 1\ ;\ 0\ 0\ 0).$$

The standard base of the mechanism constraint system is

$$\$^r_{m1} = (1\ 0\ 0\ ;\ 0\ 0\ 0)$$
$$\$^r_{m2} = (0\ 1\ 0\ ;\ 0\ 0\ 0) \tag{33}$$
$$\$^r_{m3} = (0\ 0\ 1\ ;\ 0\ 0\ 0),$$

which includes three non-coplanar constraint forces passing through a common point. To form such a mechanism constraint system, the limb constraint system may only contain one constraint force or two constraint forces or three constraint forces.

Figure 9 shows a 3-DoF 3 – $^{u1}R^{u2}P^{u1}R(^iR^jR)_N$ spherical PM, which was presented by Karouia and Hervé (2002). Counting from the base, the first revolute axis $\$_{i1}$ and the third revolute axis $\$_{i3}$ are parallel to each other and bevel with the base plane. The second prismatic pair $\$_{i2}$ is perpendicular to the two adjacent revolute axes. The fourth revolute axis $\$_{i4}$ and the fifth revolute axis $\$_{i5}$ intersect at a common point and form a 2R spherical subchain. The three limb central points coincide with each other.

The limb twist system in non-singular configuration is

$$\$_{i1} = (l_1\ m_1\ n_1\ ;\ a_1\ b_1\ c_1)$$
$$\$_{i2} = (0\ 0\ 0\ ;\ l_2\ m_2\ n_2)$$
$$\$_{i3} = (l_1\ m_1\ n_1\ ;\ a_3\ b_3\ c_3) \tag{34}$$
$$\$_{i4} = (l_4\ m_4\ n_4\ ;\ 0\ 0\ 0)$$
$$\$_{i5} = (l_5\ m_5\ n_5\ ;\ 0\ 0\ 0),$$

where

$$a_1 = y_1n_1 - z_1m_1, \quad b_1 = z_1l_1 - x_1n_1,$$
$$c_1 = x_1m_1 - y_1l_1, \quad a_3 = y_3n_1 - z_3m_1,$$
$$b_3 = z_3l_1 - x_3n_1, \quad c_3 = x_3m_1 - y_3l_1, \text{ and}$$
$$(x_1, y_1, z_1)$$

denotes a point in the first revolute axis and (x_3, y_3, z_3) denotes a point in the third revolute axis, respectively.

By calculating the screws reciprocal to the limb twist system in eq (34), we have

$$\$^r_{i1} = (l_1\ m_1\ n_1\ ;\ 0\ 0\ 0). \tag{35}$$

Equation (35) shows that such a limb exerts a constraint force on the moving platform. This constraint force passes through the mechanism central point and is parallel to the first revolute axis $\$_{i1}$.

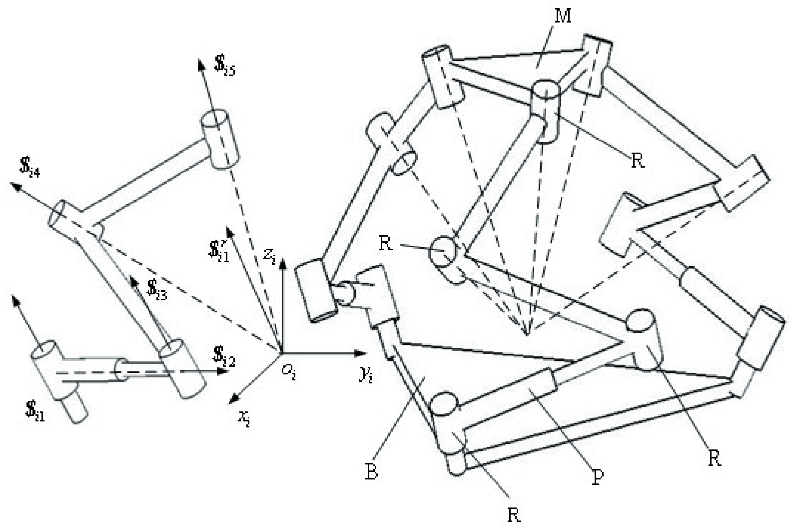

Fig. 9. 3-DoF $3 - {}^{u_1}R^{u_2}P^{u_1}R({}^{i}R^{j}R)_N$ rotational PM.

Note that the first revolute axis is actually fixed on the base, so in any non-singular configuration, the three constraint forces always intersect at the central point and are not coplanar. The standard base of the mechanism constraint system is the same as eq (33). Thus the moving platform loses all three translational DoFs and is a non-instantaneous spherical PM.

Obviously, this mechanism contains no common constraints and redundant constraints, then we have $\lambda = 0$ and $\nu = 0$.

Using eq (4), we have

$$M = 6(14 - 15 - 1) + 15 = 3. \qquad (36)$$

Another example is the 3-RRS wrist proposed by Di Gregorio (2002), which contains a spherical pair and a 2R spherical subchain.

5.2. 3-DoF PMs with Three Translational DoFs

The standard base of the mechanism twist system is

$$\begin{aligned} \$_{m1} &= (0\ 0\ 0\ ;\ 1\ 0\ 0) \\ \$_{m2} &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\ \$_{m3} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \qquad (37)$$

The standard base of the mechanism constraint system is

$$\begin{aligned} \$^r_{m1} &= (0\ 0\ 0\ ;\ 1\ 0\ 0) \\ \$^r_{m2} &= (0\ 0\ 0\ ;\ 0\ 1\ 0) \\ \$^r_{m3} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \qquad (38)$$

To form such a mechanism constraint system, the limb constraint system only contains couples and all the limb constraint couples must be non-coplanar. The limb constraint system can contain one constraint couple or two constraint couples or three constraint couples.

Figure 10 shows a $3 - {}^{x}R^{x}R^{x}R^{u}R^{u}R$ translational PM, which was presented by Frisoli et al (2000) and by Carricato and Parenti-Castelli (2001). The limb twist system is given by

$$\begin{aligned} \$_{i1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$_{i2} &= (1\ 0\ 0\ ;\ 0\ b_2\ c_2) \\ \$_{i3} &= (1\ 0\ 0\ ;\ 0\ b_3\ c_3) \\ \$_{i4} &= (0\ m_4\ n_4\ ;\ a_4\ b_4\ c_4) \\ \$_{i5} &= (0\ m_4\ n_4\ ;\ a_5\ b_5\ c_5). \end{aligned} \qquad (39)$$

The limb constraint system is given by

$$\$^r_{i1} = (0\ 0\ 0\ ;\ 0\ n_4\ \ -m_4), \qquad (40)$$

which is perpendicular to both $\$_{i3}$ and $\$_{i4}$. The three limb constraint couples $\$^r_{11}$, $\$^r_{21}$, and $\$^r_{31}$ are obviously non-coplanar in space and they form a 3-system given by eq (38). After the moving platform undergoes any finite translation, the mechanism constraint system remains unchanged. Thus, the mechanism has three translational DoFs and is not instantaneous.

This $3 - {}^{x}R^{u}P^{x}R({}^{i}R^{j}R)_N$ mechanism obviously contains no common constraints and redundant constraints, then we have $\lambda = 0$ and $\nu = 0$.

Using eq (4), we have

$$M = 6(14 - 15 - 1) + 15 = 3. \qquad (41)$$

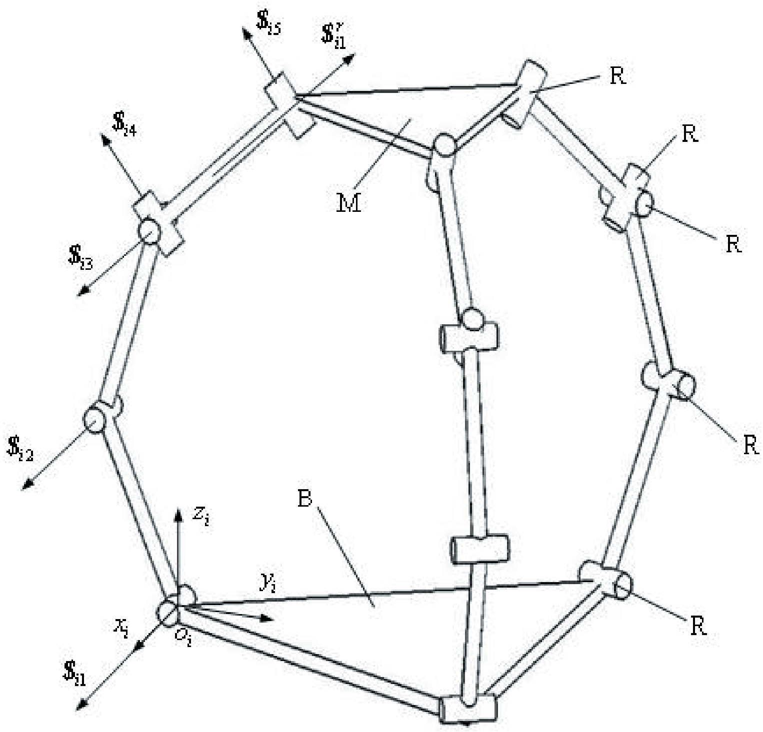

Fig. 10. 3-DoF $3-{}^{x}R^{x}R^{x}R^{u}R^{u}R$ tranlational PM.

5.3. 3-DoF PMs with Two Rotational DoFs and One Translational DoF

In this case, the mechanism constraint system must contain one couple and two forces. For simplicity, we focus on the 3-DoF PM with two rotational DoFs in the XY plane and one translational DoF along the Z-axis, denoted by $2RIT^z$. The standard base of the mechanism twist system is given by

$$\begin{aligned} \$_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$_{m3} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \tag{42}$$

The standard base of the mechanism constraint system is given by

$$\begin{aligned} \$^r_{m1} &= (1\ 0\ 0\ ;\ 0\ 0\ 0) \\ \$^r_{m2} &= (0\ 1\ 0\ ;\ 0\ 0\ 0) \\ \$^r_{m3} &= (0\ 0\ 0\ ;\ 0\ 0\ 1). \end{aligned} \tag{43}$$

When the limb kinematic chain consists of five kinematic pairs, it only exerts one constraint. If the constraint is a couple, no translations can be constrained. So the constraint must be a force and the combined effect of all the $p \cdot q$ forces must equal one couple not parallel to the XY plane and two forces parallel to the XY plane.

Figure 11 shows such a 3-DoF $3-{}^{x}R^{u}P^{x}R({}^{i}R^{j}R)_N$ PM. Note that the three limb central points form a triangle, *ABC*, which is fixed relative to the moving platform. In the initial configuration, as shown in Figure 11, the triangle *ABC* is parallel to the moving platform plane.

Note that each limb constraint force is parallel to the first revolute axis and passes through the corresponding limb central point. It is obvious that the three limb constraint forces lie in the triangle *ABC* and do not intersect at a common point. Thus, the standard base of the mechanism constraint system is the same as eq (43) and the moving platform loses two translational DoFs in the base plane and one rotational DoF about the normal of the base plane. Similarly, we can identify that this mechanism is not instantaneous.

Since there are no coaxial constraints and redundant constraints in this mechanism, we have $\lambda = 0$ and $\nu = 0$. The mobility of the $3-{}^{x}R^{u}P^{x}R({}^{i}R^{j}R)_N$ PM is also given by

$$M = 6(14 - 15 - 1) + 15 = 3. \tag{44}$$

5.4. 3-DoF PMs with One Rotational DoF and Two Translational DoFs

In this case, the mechanism constraint system should contain two couples and one force. For simplicity, we focus on the 3-DoF PM with two translational DoFs in the XY plane and one rotational DoF about the Z-axis, which is denoted by $2T^{xy}1R^z$.

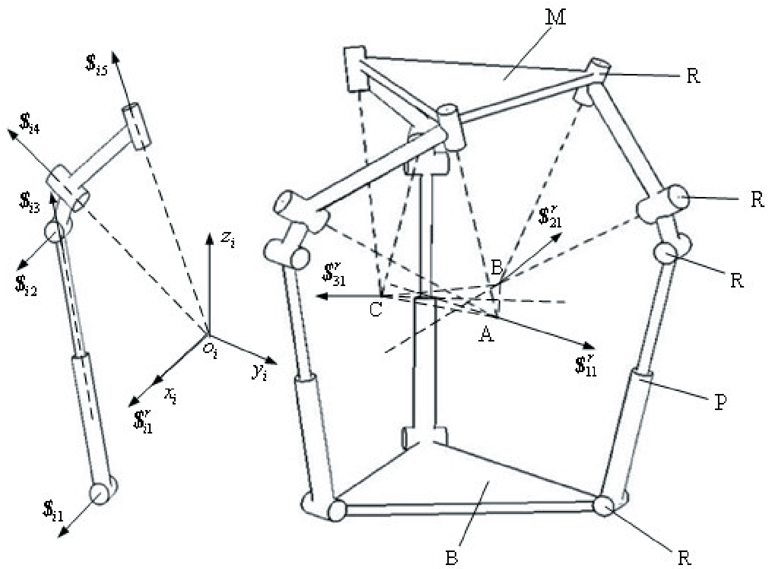

Fig. 11. $2RIT^z$ 3-DoF $3-{}^xR^uP^xR({}^iR^jR)_N$ PM.

The standard base of the mechanism twist system is given by

$$\begin{aligned}\$_{m1} &= (0\ 0\ 1\ ;\ 0\ 0\ 0)\\ \$_{m2} &= (0\ 0\ 0\ ;\ 1\ 0\ 0)\\ \$_{m3} &= (0\ 0\ 0\ ;\ 0\ 1\ 0).\end{aligned} \tag{45}$$

The standard base of the mechanism constraint system is given by

$$\begin{aligned}\$^r_{m1} &= (0\ 0\ 1\ ;\ 0\ 0\ 0)\\ \$^r_{m2} &= (0\ 0\ 0\ ;\ 1\ 0\ 0)\\ \$^r_{m3} &= (0\ 0\ 0\ ;\ 0\ 1\ 0).\end{aligned} \tag{46}$$

When the limb kinematic chain consists of five kinematic pairs, it only exerts one constraint. If the constraint is a couple, no translations can be constrained. So the constraint must be a force and the combined effect of all the $p \cdot q$ forces must equal two couples and one force.

Figure 12 shows a 3-DoF $3-{}^zR^zR^zR({}^iR^jR)_N$ PM. The three limb central points also form a triangle, which is fixed relative to the moving platform and is parallel to the moving platform plane. Note that each limb constraint force is parallel to the first revolute axis and passes through the corresponding limb central point. It is obvious that the three limb constraint forces are parallel in space. Thus, the standard base of the mechanism constraint system is the same as eq (46) and the moving platform loses one translational DoF in the Z-axis and two rotational DoFs in the XY plane.

Since there are no coaxial constraints and redundant constraints in this mechanism, we have $\lambda = 0$ and $\nu = 0$. The mobility of the $3-{}^xR^uP^xR({}^iR^jR)_N$ PM is also given by

$$M = 6(14 - 15 - 1) + 15 = 3. \tag{47}$$

6. Conclusions

1. The identification of common constraints and redundant constraints by constraint analysis is effective and simple. Based on this, the general Grübler-Kutzbach criterion can be correctly applied to the lower-mobility PM. The concept of a generalized pair is useful in the mobility analysis of PMs containing closed loops in the limb.
2. The characteristics of constraint and structure of 4-DoF and 5-DoF symmetrical PMs are revealed, which is useful both in type synthesis of lower-mobility PMs and for understanding the constraint-synthesis method.
3. It is shown in this paper how to synthesize limb kinematic chains and mechanisms by the constraint-synthesis method. The constraint-synthesis method proves a powerful tool for type synthesis of lower-mobility PMs and is of considerable importance in mobility analysis.
4. Many novel lower-mobility PMs are proposed for the first time, including systematic enumerations of symmetrical 4-DoF and 5-DoF PMs.

Appendix

In Tables A1–A3, g/p denotes the number of kinematic pairs in a single limb in a symmetrical lower-mobility PM.

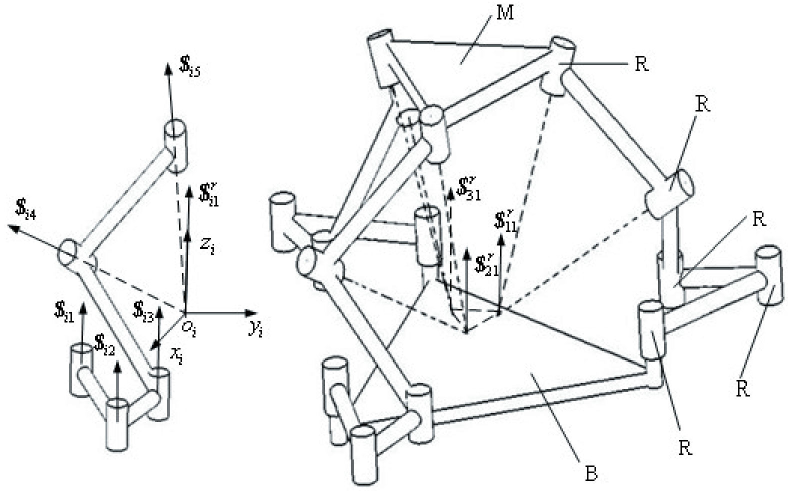

Fig. 12. $2T^{xy}1R^{z}$ 3-DoF $3-{}^{z}R^{z}R^{z}R({}^{i}R^{j}R)_{N}$ PM.

Table A1. Symmetrical $3R2T^{xy}$ 5-DoF PMs

g/p	Constraint	Structure
5	• Each limb exerts one constraint force on the moving platform. • All the limb constraint forces must be coaxial.	• The limb kinematic chain must contain a 2R or 3R spherical subchain. • The prismatic pair must be parallel to the base plane. • The revolute pairs except those in the 2R or 3R subchains must be perpendicular to the base plane. • All the limb central points must coincide with one another and no spherical pairs exist. • All the 2R or 3R subchains must connect either to the base or the moving platform simultaneously.

Table A2. Symmetrical 2R3T 5-DoF PM

g/p	Constraint	Structure
5	• Each limb exerts a constraint couple on the moving platform. • All the axes of the limb constraint couples must be parallel.	• The limb kinematic chain contains two revolute pairs at least. • The revolute pairs fall into two groups. The axes of one group are parallel to the axis of one desired rotational DoF. The axes of the other group are parallel to the axis of the other desired rotational DoF. • In all limbs, the revolute axes most adjacent to the base must be set parallel to one another and the revolute axes most adjacent to the moving platform must be set parallel to one another. • No spherical pairs exist in the limb kinematic chain.

Table A3. Symmetrical 3T1R^z 4-DoF PM

g/p	Constraint	Structure
4	• Each limb exerts two constraint couples on the moving platform. • All the limb constraint couples must be perpendicular to the Z-axis.	• The limb kinematic chain only contains prismatic pairs and revolute pairs. • The axes of revolute pairs must be perpendicular to the base plane.
5	• Each limb exerts one constraint couple on the moving platform. • All the limb constraint couples must be perpendicular to the Z-axis.	• The limb kinematic chain contains no spherical pairs. • The revolute pairs fall into two groups. The axes of one group are parallel to the base plane. The axes of the other group are perpendicular to the base plane. • The revolute pair axes parallel to the base plane in each limb must not be parallel to one another. • The revolute pair whose axes are perpendicular to the base plane must connect either to the base or the moving platform simultaneously.

Table A4. Symmetrical 3RIT^z 4-DoF PM

g/p	Constraint	Structure
4	• Each limb exerts two constraint forces on the moving platform. • All the limb constraint forces must be coplanar and intersect at a common point.	• The limb kinematic chain must contain a 3R spherical subchain. • The prismatic pair must be parallel to the Z-axis. • All the limb central points must coincide with one another and no spherical pairs exist. • All the 3R subchains must connect either to the base or the moving platform simultaneously.
5	• Each limb exerts one constraint force on the moving platform. • All the limb constraint forces must be coplanar and intersect at a common point.	• The limb kinematic chain must contain a 2R or 3R spherical subchain. • When prismatic pairs exist, at least one prismatic pair must not be parallel to the base plane. • The revolute pair axes except those in the 2R or 3R subchains must be parallel to the base plane. • All the limb central points must coincide with one another and no spherical pairs exist. • All the 2R or 3R subchains must connect either to the base or the moving platform simultaneously.

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