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Constraint Singularities of Parallel Mechanisms

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Abstract

The concept of *constraint singularity* is introduced. This is a phenomenon occurring in parallel mechanisms with reduced freedoms when the screw system, formed by the constraint wrenches in all legs, loses rank.

1 Introduction

In the recent robotics literature, there has been increasing interest in spatial parallel manipulators (PMs) with fewer than six degrees of freedom. It is hoped that such mechanisms can perform successfully many tasks that have so far required 6-dof platforms and achieve lower device and operation costs, due to simplified designs involving fewer links and actuators. Furthermore, architectures where some end-effector freedoms have been eliminated can offer a larger workspace for the remaining platform motions.

The intended advantages of reduced-freedom manipulators are best achieved in kinematic chains without passive legs where unwanted freedoms of the platform are constrained by *each* leg of the manipulator. This allows to achieve both the desired design simplification and the vital increase in the remaining motion range.

In the classical cases of planar and spherical mechanisms, each leg has the same freedoms as the whole mechanism. In contrast, in a constrained PM different leg chains impose different constraints and only their combined effect leads to the desired end-effector freedoms. In the best known mechanisms of this kind, there are three 5-dof legs, each imposing one constraint on the platform. As a result, the platform (and the whole mechanism) has 3 dof.

The kinematic analysis of constrained PMs and, presents interesting new challenges. In this paper, we focus on one such problem. We examine a particular kind of singularity, which we call *constraint singularity* and which occurs only in constrained PMs. Several researchers have come across such configurations while analyzing specific mechanisms. We show that each of those separate occurrences is a manifestation of the same kinematic phenomenon. We define constraint singularity in general terms, examine its causes and effects and outline some pitfalls that may cause an analyst to overlook such a configuration.

2 Freedoms, Constraints and Constraint Singularity

Consider a general PM with legs labeled by the letters $P = A, B, \ldots$. The leg chain P has n_P joints. The relationship between the instantaneous motion of the platform, the *output twist* $\boldsymbol{\xi} = (\boldsymbol{\omega}, \mathbf{v})$, and the joint velocities $\dot{\theta}_i^P$ is given by the twist equations of the serial subchains:

$$\boldsymbol{\xi} = \sum_{i=1}^{n_P} \dot{\theta}_i^P \boldsymbol{\xi}_i^P, \quad P = A, B, \ldots, \tag{1}$$

where the joint screws are denoted by $\boldsymbol{\xi}_i^P$.
Equation (1) is a necessary and sufficient condition for the output twist, $\boldsymbol{\xi}$, and the joint velocities, $\dot{\theta}_i^P$, to be feasible. Therefore, the space of all possible platform twists, $\mathcal{T}$, i.e., the system of *platform freedoms* is given by

$$\mathcal{T} = \bigcap_{P=A,B,\ldots} \mathcal{T}_P, \tag{2}$$

where $\mathcal{T}_P$ is the output twist space of the subchain P,

$$\mathcal{T}_P = \mathrm{Span}\,(\boldsymbol{\xi}_1^P, \ldots, \boldsymbol{\xi}_{n_P}^P), \quad P = A, B \ldots. \tag{3}$$

For a twist(wrench) system $\mathcal{L}$, let $\mathcal{L}^\perp$ be its reciprocal wrench(twist) system, $\mathcal{L}^\perp = \{\boldsymbol{\rho} \mid \boldsymbol{\rho} \circ \boldsymbol{\xi} = 0 \;\; \forall \boldsymbol{\xi} \in \mathcal{L}\}$, where "$\circ$" is the reciprocal screw product.

Physically, the reciprocal screws in $\mathcal{W}_P = \mathcal{T}_P^\perp$ represent wrenches which, if applied to the platform, can be resisted by the leg P with zero torque from the actuators, i.e., the wrenches of constraint imposed by the leg, and $\mathcal{W} = \mathcal{T}^\perp$ is the system of *platform constraints.* Since $(\mathcal{L}^\perp)^\perp = \mathcal{L}$, Equation (2) implies

$$\mathcal{T}^\perp = \sum_{P=A,B,\ldots}^{D} \mathcal{T}_P^\perp. \tag{4}$$

Therefore, for any configuration, $\dim \mathcal{W} + \dim \mathcal{T} = 6$

In all nonsingular configurations, $\dim \mathcal{W} = 6 - n$, where n is the global (full-cycle) mobility of the mechanism. In a singular configuration, the constraint system of a leg, $\mathcal{W}_P$, can only increase in dimension. However, due to intersections of the leg constraint systems it is possible for the platform constraint system, $\mathcal{W}$, to lose rank and

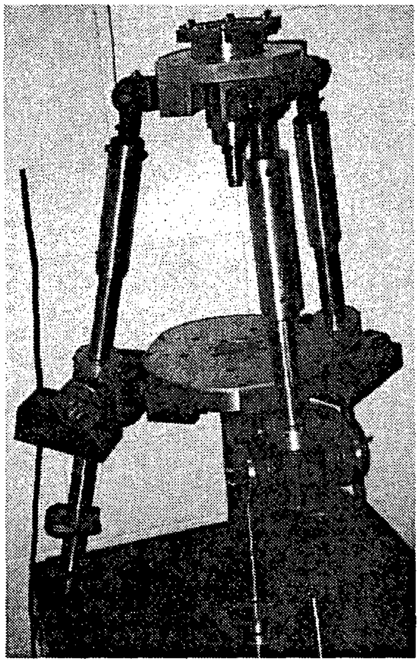

Figure 1: The 3-*UPU* mechanism at SNU in a singular configuration (courtesy of Prof. Frank Park)

its dimension to become lower than $6-n$. If this occurs we will say that the mechanism is in a *constraint singularity.* It immediately follows that in such a configuration the freedom system of the platform, $\mathcal{T}$, will be of dimension higher than n, and therefore the instantaneous mobility of the mechanism is higher than its full-cycle mobility.

3 Rotation and Translation Singularities

Possibly the most popular type of constrained PMs are positioning devices with three translational degrees of freedom. The motivation for the present work grew from a singular configuration of one such translational manipulator (Fig. 1). The mechanism was built by the team of Prof. F.C. Park at Seoul National University. It is a 3-*UPU* architecture. In a configuration like the one shown, where all legs have equal length and the platform is directly above the base, the platform is highly unstable even when all three prismatic actuators are locked.

This singularity has puzzled many analysts since the 3×3 Jacobian matrix of the input-output velocity equation is not singular. For a translational parallel robot with three legs the relationship between the platform velocity, $\mathbf{v}$, and the 3-vector of the active-joint rates, $\dot{\boldsymbol{\theta}}_a$ is usually described by a 3-dimensional linear equation:

$$\mathbf{Z}\mathbf{v} = \boldsymbol{\Lambda}\dot{\boldsymbol{\theta}}_a\,. \tag{5}$$

What is usually understood when such a mechanism is said to be in a singularity is a configuration where the matrix $\mathbf{Z}$ becomes singular and, hence, there is uncontrollable motion of the platform even when the input velocities are zero. (This is referred to as a singularity of Type 2 in [1] and RO (redundant output) type in [2]). However, in the discussed configurations, the row vectors of $\mathbf{Z}$ (which, in this case, are simply the unit vectors along the legs) are linearly independent. In fact, it is possible that the three legs are at right angles with each other and then the configuration could be termed "isotropic."

This apparent contradiction could be explained by noting that the singularity of $\mathbf{Z}$ would mean that there is an uncontrollable *translation*, i.e., that the platform can translate even with locked actuators. In the studied configuration, when the actuators are locked, the platform cannot translate, therefore, there is no classic parallel singularity. But the platform is unstable and can move. The reason is that, in this configuration, the PM ceases to be translational and the platform can rotate. The mechanism has gained a new *additional* rotational freedom.

A PM is translational only when the system of the constraining wrenches, $\mathcal{W}$, contains as a subspace the 3-system of all pure moments. Then its reciprocal system, the freedoms system $\mathcal{T}$, must contain *only* translations. In a 3-*UPU* manipulator, as long as the two *U*-joints in a leg are parallel, the leg constraint system $\mathcal{W}_P$ contains a pure moment perpendicular to the *U*-joint plane. The constraint system $\mathcal{W}$ will contain the three moments of the three legs. When the three moments of the three legs are linearly independent all pure moments will be in $\mathcal{W}$.

However, in the studied configuration, the planes of all *U*-joints are parallel. The constraining moments of all three legs are identical and the systems $\mathcal{W}_P$ are the same 1-system. Then, $\mathcal{W}$ consists of only one screw — the vertical moment. Hence, the twist system of the platform is a five-system and, since the mechanism has five instantaneous dof, it is no longer translational.

To identify the exact nature of the instantaneous self-motion when the actuators are locked, we need to find the constraining wrenches of the same mechanism but as if there were no prismatic joints. Now, the leg constraints form a 2-system spanned by the vertical moment and a pure force along the leg. The platform constraints then form the 4-system spanned by all forces passing through the point at which all three legs intersect, as well as the pure moment about the vertical direction. Hence, the motion system, $\mathcal{T}$, is the 2-system of all rotations with horizontal axes through the legs' intersection point.

The ability of some translational manipulators to allow rotations in certain configurations was discovered by Di Gregorio and Parenti-Castelli [3]. However, much earlier, in a study of shaft couplings [4], Hunt already described

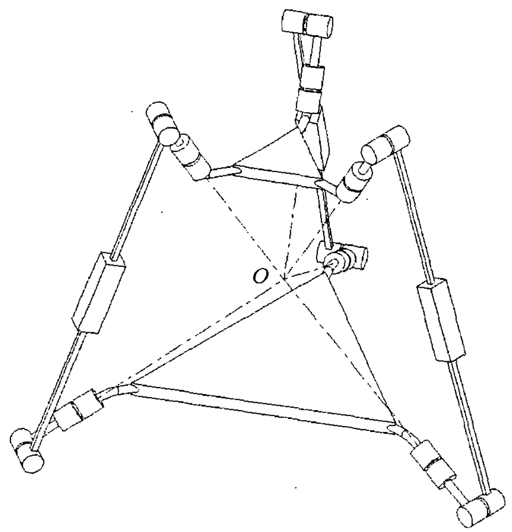

Figure 2: A 3-*UPU* wrist in a constraint singularity

a 3-*URU* kinematic chain, equivalent to the one on Fig. 1, and identified the constraint singularities discussed above as uncertainty configurations, i.e. singularities with increased instantaneous mobility (IIM). In [5] the authors analyze the singularities of a general 3-*UPU* translational mechanism assembled for translation, i.e., satisfying the conditions given in [6]. They express the (angular and linear) velocity of the platform as linear functions of *all* joint velocities, active and passive, and find the conditions for a non-zero instantaneous motion to be possible when the actuator velocities are all zero. After studying the linear system of equations, the authors observe that all conditions for a non-zero platform motion can be separated into conditions for either a redundant translation or an uncontrolled rotation of the end effector. They call the two groups of configurations, *translational* and *rotational* singularities, respectively.

It is important to realize that the phenomenon of constraint singularity is more general and more fundamental than the ability of certain translational robots to rotate. To make this clear, let us look at an example of a constraint singularity that can also be described as a translation singularity. Consider another 3-*UPU* mechanism, this time assembled not as a translational device but as a spherical wrist (Fig. 2). The design is such that all three R-joint axes in the base have a common intersection point; the same is true for the three platform R joints; and, finally, these two points are made to coincide. This mechanism was first described in [7] and analyzed in [8].

The constraint wrench for each leg is a pure force through the rotation centre parallel to the two R joints fixed in the leg. Normally, these three forces are linearly independent. The constraint system is then composed of all forces through the centre and hence the system of platform freedoms is the reciprocal screw system, namely, all rotations about the point O. However, if the three leg planes can intersect in a common line, as is the case in Fig. 2, the three forces become linearly dependent. Now the constraint system is only two-dimensional and its reciprocal system, that of the platform freedoms, becomes four-dimensional. And since the constraint system is a planar pencil of rotations, the platform freedom system is spanned by a translation normal to the plane of the pencil in addition to all the rotations through O. Thus, the end effector gains a translational degree of freedom.

For this mechanism, the "rotation singularities" are the usual parallel-chain singularities (Type 2, RO) where no extra freedom is gained but one of the existing platform freedoms becomes uncontrollable. For our purposes, we will assume that the actuated joints are the revolute joints at the base. (We will look at what happens when the prismatic joints are actuated later in Section VII). In this case, for an RO singularity to occur, the three planes of the upper U joints must intersect in a common line, i.e., they must have linearly dependent normal vectors. It is no accident that this is similar to the condition for the rotational singularities of the 3-*UPU* mechanism studied in [5]. The condition for the existence of an instantaneous self motion when the actuators of the robot in Fig. 2 are locked decomposes in two just as is the case for a translational robot. Indeed, the constraint system with the actuators locked is decomposed into two systems: a system of forces through the rotation centre; and a system of moments normal to the upper U-joint planes. The two systems cannot intersect, so singularity occurs when one of them loses rank. In one case the platform has an uncontrollable rotation, in the other there is a translation unstoppable with the actuators. It is the translational singularity that is a constraint singularity, while the rotational one is of the more usual RO type.

The fundamental difference between the two groups of singularities is not that the "gained" freedom is a rotation in one case and a translation in the other. Rather, the uncontrollable motion is a truly new one at a constraint singularity, and one of the already present freedoms at a usual parallel singularity.

4 Singularities of 3-*RPS* Manipulators

To see more clearly that the rotation-translation division is beside the point, let us look at another PM, a 3-*RPS* mechanism and two of its singular configurations.

The configuration in Fig. 3, where one R-joint axis is in the plane of the platform, is an ordinary RO-type singularity. The platform has one uncontrollable freedom, namely a rotation about the line BC, when the prismatic actuators are locked. In Fig. 4 we see another singular

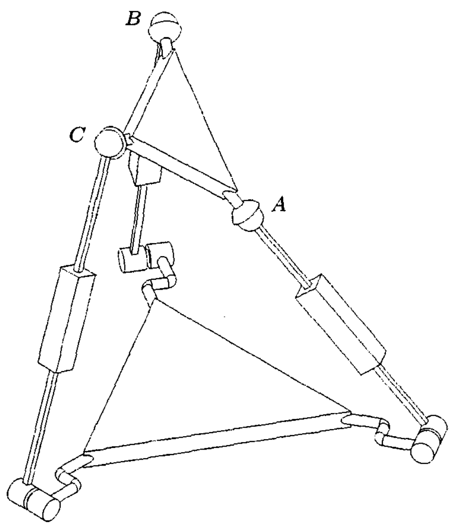

Figure 3: An RO-type singularity of a 3-*RPS* mechanism

configuration. Here, the uncontrollable motion is again a rotation, with an axis passing through both point C and the intersection point of legs A and B. The second configuration is, in fact, a constraint singularity.

Despite the apparent similarity, the two singularities are quite different. In Fig. 3 the constraint wrenches (forces through the S joints and parallel to the R joints) are linearly independent and the platform has three dof. In Fig. 4, however, the three constraint wrenches all pass through point C and lie in the platform plane, and hence form only a two-system. The platform gains a freedom and can instantaneously move with 4 dof. Indeed, it can rotate arbitrarily about point C and translate vertically.

From a kinematic point of view these are completely different phenomena. In one case, the loss of control is due to the (so to speak) unfortunate choice of the actuators. If, in Fig. 3, you actuate the base R joint rather than the P joint of leg B, you would have full control of the platform. In Fig. 4, there is no choice of three actuators that will prevent the platform from moving instantaneously.

Let us identify all constraint singularities. Three non-parallel zero-pitch screws are linearly dependent only if they span a planar pencil. Since the constraint forces are horizontal, it follows that the platform and base must be parallel. Moreover, it can be shown that the three force axes in the horizontal platform plane can intersect in one point, H, only if the platform is upside down as in Fig. 4. The feasible poses of the platform in every horizontal plane form a one-parameter set obtained by rolling the circle, c_m, drawn around the platform triangle, on the inside of a fixed circle, c_f, twice the radius of c_m and centred at G, right above the base centre, Fig. 5.

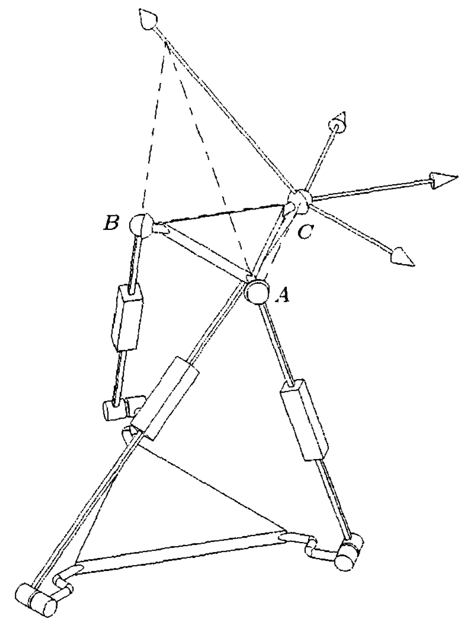

Figure 4: A constraint singularity of a 3-*RPS* mechanism

The configuration in Fig. 4 is the special case when H coincides with a platform vertex. If this is not the case, the redundant motion with locked actuators will not be a pure rotation as in Fig. 4, but rather a screw motion, more precisely a linear combination of a rotation through H and the vertical translation. We are not aware of any previous publication of similar results for the 3-*RPS* chain. However, in [9] there is mention of uncertainty configurations of a 3-*PSP* chain, also with an upside-down platform.

The distinction between the two types is important also because a constraint singularity, whether it is rotational or not, will generally fail to be detected by studying the input-output equations.

Finally, we show another constraint singularity of a (different) 3-*RPS* mechanism (Fig. 6). This time, the platform is in a much more unperturbed pose than in Fig. 4, however, the three base R joints need to intersect in a point. The platform can instantaneously yaw (rotate about its normal axis) with locked actuators. This configuration was described in [10].

5 On Singularity Identification

The velocity kinematics of a n-dof parallel manipulator is most conveniently described with an n dimensional equation like (5), at least for most of their configurations [1], [11]. As we mentioned in Section III, this equation can be used to detect RO-type singularities. However, it must be emphasized that this equation cannot be used to identify

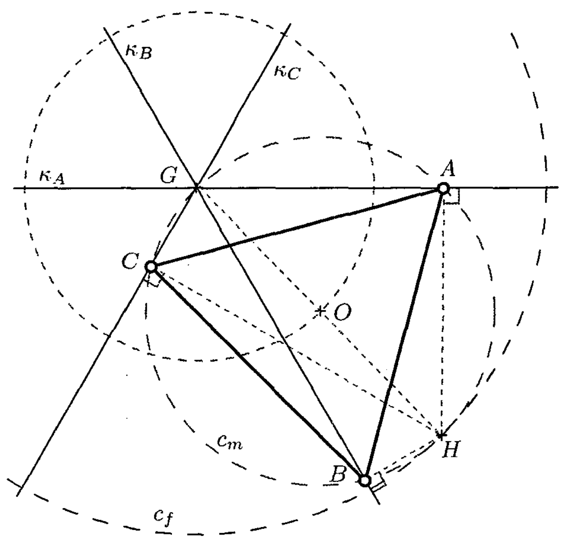

Figure 5: Constraint singularities of a 3-*RPS* mechanism

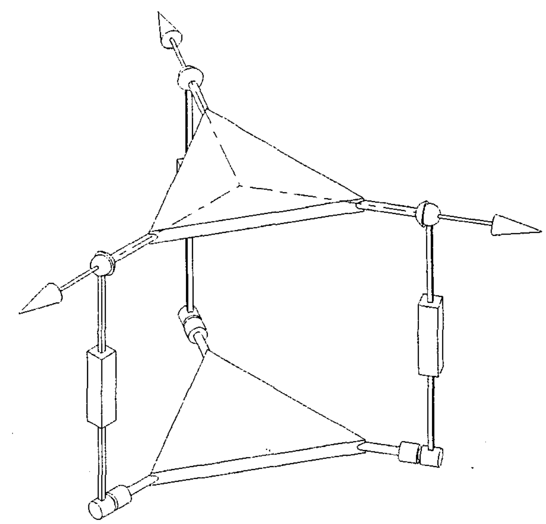

Figure 6: A 3-*RPS* PM in a constraint singularity

constraint singularities. Moreover, the equation no longer describes the velocity kinematics at a constraint singularity. Therefore, one needs to check for constraint singularities before eliminating the passive velocities and performing the the input-output velocity analysis. Unlike others, Di Gregorio and Parenti-Castelli did not miss these singularities because they used the full velocity equations where passive velocities were present.

The fact that the singularities of a mechanism with closed kinematic loops cannot be properly described and understood if only input-output equations are used was emphasized in [2], where singularities were defined and classified using the general velocity equation of an arbitrary mechanism. For some parallel mechanisms, a careful selection of the input-output equations allows comprehensive singularity analysis [11]. These are mechanisms where the mobility of the serial chain of each leg is the same as that of the mechanism as a whole, i.e., not of the constraint type discussed in the present paper. However, even for the mechanisms discussed in [11], there is one singularity type that cannot be described with the input-output equation alone, namely IIM (increased instantaneous mobility) also referred to as an uncertainty configuration by Hunt (for single-loop chains) [12].

It must be underlined that the identification and proper interpretation of constraint singularities is greatly facilitated when geometrical methods, such as screw theory are used. Some of the conditions for singularity can be very easily formulated in geometrical terms while their symbolic derivation is quite complicated. A good example is the condition for constraint singularity of 3-*UPU* manipulators, calculated symbolically in [5]. Screw geometry, as we showed in Section III, gives the condition without any computation: the three normal vectors to the *U*-joint planes in the three legs must be linearly dependent. It should be noted that the knowledge of the geometrical properties of screw systems (as classified first in [12]) is of special value in the analysis.

6 IIM-type and Constraint Singularities

We pointed out that constraint singularities are in fact IIM singularities (uncertainty configurations). Indeed, assuming that the mechanism is non-redundant, its full-cycle mobility is equal to the mobility of the end-effector. Yet, not every IIM configuration is a constraint singularity since it is possible for the mechanism to gain a freedom while the dof of the end-effector remains unchanged.

To illustrate this and simultaneously provide what must be the simplest example of a constraint singularity, let us consider the four-bar linkage in Fig. 7(a). Assume that the link lengths are such that the mechanism can become flattened, Fig. 7(b). In the shown configuration, the kinematic chain as a whole acquires an instantaneous mobility of two. Usually the output link of a four bar is one of the rotors and, therefore, the end-effector will always remain with at most one dof, including in Fig. 7(b).

However, let us assume that the output link is the coupler, a kind of "platform" of a 1-dof planar "parallel manipulator" with two *RR* legs, one of which is passive. Now, in the singularity of Fig. 7(b) the end-effector gains a freedom and has two instantaneous dof. Indeed, it can now rotate about any point on the line containing all joint centres and, of course, translate in the normal direction.

To see that this is a constraint singularity like the ones

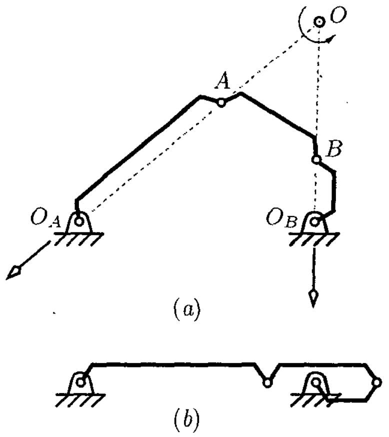

Figure 7: A flattened four-bar mechanism in (a) a nonsingular configuration and (b) a constraint singularity

already described, observe that each RR leg imposes one planar constraint on the platform: a force through the two joint centres of the leg. Normally, these two lines are distinct and span a 2-system. The planar component of the reciprocal system is the 1-dof system of the coupler freedoms. When the lines of the two constraint forces intersect, the freedoms are the rotations about the intersection point, Fig. 7(a). When the two lines are parallel the coupler must instantaneously translate in the normal direction. However, in Fig. 7(b) the constraints become one, more precisely they are linearly dependent and span only a one system. The system of output motions then becomes the two-system which we already described.

A further exploration of constraint singularities as IIM-type singularities [13], as well as an earlier version of the present study is available at the Para*ll*eMIC site.

7 Added and Uncontrollable Freedoms

We saw that at a constraint singularity the platform acquires additional dof. Then, obviously, not all of the output freedoms can be controlled. However, this does not mean that it is always a new degree of freedom (i.e., one that exists due to the constraint singularity) that will be uncontrollable. More precisely, the redundant end-effector motion present when the actuators are locked may be inside the screw system of platform freedoms which exist at a non-singular configuration.

For example, consider the PM in Fig. 2, but this time, with prismatic actuators. As we pointed out, the shown configuration is a constraint singularity and the platform gains a translational dof. Yet, when the prismatic actuators are locked the end-effector cannot translate. The only instantaneously possible motion is a rotation about a vertical axis through the centre. The translation of the platform can be achieved but only with the actuators.

While the system of uncontrollable freedoms is well defined—it is given by the motions possible when the actuators are locked—a system of "controllable freedoms" cannot be specified in a unique way. It is useful to define a screw system as controllable if it has a zero intersection with the uncontrollable system and together with it spans the whole of $\mathcal{T}$. Note that we cannot ensure that the output twist is restricted to such a system, due to the possible presence of transversal redundant motion (so, in practice, we do not have full control).

For example, assuming prismatic actuators in Fig. 2, the system spanned by the horizontal rotations through O and the vertical translation is controllable, in the defined sense. Yet, we cannot enforce a motion in that system since we cannot prevent the platform to also rotate about the vertical axis. Similarly, for a 3-*UPU* translational PM at a constraint singularity we will say that the system of platform translations is controllable, although we cannot actually control the linear velocity of all points of the body. In both examples, the mentioned controllable systems are not unique. In fact, any motion that is not a RO motion can be part of a controllable system.

This understanding on the meaning of the terms controllable and uncontrollable, when referring to the possible platform motions, will allow us to make a clear distinctions between the two constraint singularities illustrated by Fig. 2 (for the two different choices of the actuated joints). In both cases we have a redundant motion of the platform, but only in one case can we say that we have a redundant *output* motion. That is so since for a rotational device, which the mechanism is intended to be, the output motion is not the motion of the platform in general, but rather the rotation of the platform. Hence, in one case, when the actuators are prismatic, we have a constraint singularity that leads to redundant output, i.e., one of the desired rotational freedoms is uncontrollable. In the other case, when the first revolute joints are the active ones, the output rotations are a controllable system. Therefore, rigorously speaking, there is no redundant output motion, and the uncontrollable translation of the platform can be thought of as a redundant *passive* motion. Similarly, in the rotational singularities of translational robots we should not speak of the uncontrollable rotation as a redundant output motion, since rotations were never intended as output, but rather as an instantaneous passive motion in the mechanism.

8 Conclusions

A constraint singularity is a configuration of a parallel mechanism with a dof of the platform, n, lower than the number of joints in any leg. At such a singularity, the screw system of the constraint wrenches degenerates and becomes of a dimension lower than $6-n$. As a result, the system of output freedoms instantaneously increases its dimension. Hence, both the mechanism as a whole and the platform have at least $n+1$ dof. The extra freedom of the platform may or may not be controllable by the actuators. Such configurations need to be identified before input-output velocity analysis is performed.

Acknowledgments

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